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PREFATORY REMARKS.

The object of the following pages has been to facilitate the acquisition of Arithmetic, by rendering its principles more simple, and easy of comprehension by the student.

In the execution of this object, the design has been to adhere strictly to the order of nature in the arrangement of the several rules—clearly to exhibit the nature, powers, and operations of numbers—the similarity and intimate relations between rules distinguished by different names, but in reality of similar import; and also the *why's* and *wherefore's*, generally, that such effects should necessarily result.

The subject of notation and numeration has here received more attention than what has been usual by authors. This has been occasioned from a full conviction, that enough had not been written or exemplified, to furnish the learner with sufficiently clear and adequate ideas to prepare him to enter, with profit, on the subsequent rules. The collocation of decimal notation, and numeration, in connection with that of whole numbers, will be obvious from the consideration, that without some knowledge of the punctuation of decimals, the student would be wholly incompetent to the management of federal money, in the ground rules of Arithmetic.

The rules of increase and decrease, with their respective subdivisions, are newly arranged. The object manifestly is, to render more clear and intelligible to the learner, the nature and similarity of rules which retain entirely different names. In this manner the learner will more obviously discover, that the nature and principles of the rules of addition and multipli-

cation must necessarily produce one and the same result; viz. they both inevitably produce an increased quantity; and that the latter rule is only a contracted method of executing the work of the former: in like manner also of decrease, viz. subtraction and division, both lessen or diminish the quantity; and the latter rule is only a contraction of the work of the former.

In the change of currencies and the reduction of coins, methods have been adopted, probably in some respects new, yet designed to simplify the labour; and by illustrating the principles of the various changes, to preclude the necessity of burdening the memory with the necessary retention of certain given numbers.

Proportion in general is placed before the rule of three; and the nature of proportion, with the distinctions between arithmetical and geometrical proportions and progressions, is carefully distinguished. This appeared to be indispensably necessary, from the consideration, that the rule of three, and most of the subsequent rules in the system of arithmetic are founded exclusively on the principles of proportion. Hence without any discriminating ideas of these important principles, the student would be left in much perplexity relative to any proper solutions of the nature and principles of all the various rules which have proportion for their basis.

The rules for interest have been pursued farther than is usual, and extended to a great variety of forms, so as to be rendered applicable to all the various business transactions, in which premiums on loans, commissions, &c. &c. require computations. Many rules are given and exemplified, some of which have probably never before been published.

The subject of foreign exchange has been enlarged upon, and the moneys of most of the commercial places on both continents have been given, to aid accountants in the more ready discharge of their labours.

Mensuration, ship's tonage and cask gauging have been appended to the work, for the purpose of embracing, very extensively, the different methods requisite in the various arithmetical calculations, and thereby to render the system of arithmetic the more complete.

The design of the questions placed under the several rules is, that the knowledge of the pupil may be easily tested, in the various rules, as he gradually advances. If, when he has passed over a rule, he can readily answer the questions under it, it is evident he thoroughly understands the nature and import of the rule; and that no seeming labour was required on his part to solve each question: on the contrary, if the questions become irksome to the student, he at once betrays his ignorance of the nature and design of the rule, and requires to be put to the im mediate revision of it.

Brevity has been consulted under each rule, so far as was deemed consistent, and still furnish a clear conception of the nature and principles of each respective rule. It is not the quantity of examples under each rule which is the great desideratum, but the clear elucidation of the rule by explanations, and a few examples, which render the principles of the rule clearly comprehensible by the student.

It appeared necessary also, that the treatise should be full and perspicuous, and at the same time evade, if possible, the imputation of its being too voluminous for a ready and practical elementary work This consideration has operated to neglect the solutions of several rules, as necessarily occupying too much space, together with the consideration, that comparatively very few regard, with any care or attention, the solutions referred to, viz. those relating to the different roots. Yet should the size not be considered objectionable, and the work merit sufficient patronage, these omissions, possibly, may hereafter be inserted.

The multiplicity of rules and notes of illustration which are given in this work, does not necessarily imply, that the learner must consequently commit the whole to memory. The object is, *to elicit ideas* in the minds of learners, and to furnish them with clear conceptions of elementary principles, which, when once clearly conceived, they can readily communicate to others, dressed, if requisite, in their own language, yet fully expressive of the true meaning and import of the rules. In this view, there is no task in really committing to memory; for it is not committing simply a phraseology of words to them of no meaning,

but it is the clear reception of the ideas, which are thus clothed with words. Still more absurd is it, that youth should be required to work out examples, without any requisition of the rules, either in their *principles* or *phraseology*; for in such a case the examples can have no application, in their estimation, to any supposed rule whatever. Hence their time and labour are lost, and worse than lost, for the want of proper direction and cultivation.

Long experience in the instruction of youth, has clearly evinced the indispensable necessity of early inculcating elementary principles upon the minds of youth. This lays a foundation for the regular growth and enlargement of their mental powers; and such early render themselves distinguished proficients in the various branches of science to which they devote attention.

It was the apparent necessities of the youth which influenced the author to engage in this work. Gladly would he contribute his feeble effort to aid "in the removal of the rubbish, that the skill of the polisher may not only cause the surface to shine," but also exhibit to view "the latent beauties and qualities" of the mind in a more conspicuous manner.

The work, such as it is, is now presented before the public, and submitted to the decisions of the candid, the judicious, and impartial.

THE AUTHOR.

New-York, August 1st, 1828.

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Pound Cost [illegible]

[illegible]

Troy Weight.

24 grains,	(marked *gr.*)	make	1 pennyweight,
20 pennyweights,	(do. *pwt.*)		1 ounce,
12 ounces	(do. *oz.*)		1 pound,
pound	(do. ℔.)		

By this weight, jewels, gold, silver, and liquors are weighed.

Avoirdupois Weight.

16 drams,	(marked *dr.*)	make	1 ounce,
16 ounces,	(do. *oz.*)		1 pound,
28 pounds,	(do. ℔.)		1 quarter of Cwt.
4 quarters,	(do. *qr.*)		1 Cwt.
20 hundred weight,	(do. *Cwt.*)		1 Ton,
Ton	(do. *T.*)		

By this weight, all coarse and drossy goods, grocery wares, and all metals, except gold and silver, are weighed.

Apothecaries' Weight.

20 grains,	(marked *gr.*)	make	1 scruple,
3 scruples,	(do. ℈,)		1 dram,
8 drams,	(do. ʒ,)		1 ounce,
12 ounces,	(do. ℥,)		1 pound,
pound	(do. ℔.)		

Apothecaries use this weight in compounding medicines.

Cloth Measure.

2¼ inches,	(marked *in.*)	make	1 nail,
4 nails,	(do. *na.*)		1 quarter of a yard,
4 quarters,	(do. *qrs.*)		1 yard,
3 quarters,			1 ell Flemish, *E. Fl.*
5 quarters,			1 ell English, *E. E.*
6 quarters,			1 ell French, *E. Fr.*

This measure is used for cloths, tapes, &c.

Dry Measure.

2 pints,	(marked *pt.*)	make	1 quart,
2 quarts,	(do. *qt.*)		1 pottle,
4 quarts,			1 half peck, or gallon,
8 quarts,			1 peck,
4 pecks,	(do. *pc.*)		1 bushel,
8 bushels,	(do. *bu.*)		1 quarter,
36 bushels,			1 chaldron,
5 quarters, or 40 bushels,			1 wey,
2 weys, or 80 bushels,			1 last.

This measure is used for grain, salt, fruits, coal, &c.

Long Measure.

3 barley corns	(marked *b. c.*)	make	1 inch,
12 inches,	(do. *in.*)		1 foot,
3 feet,	(do. *ft.*)		1 yard,
16½ feet, or 5½ yards	(do. *yd.*)		1 rod, pole, or perch,
40 poles,	(do. *pol.*)		1 furlong,
8 furlongs,	(do. *fur.*)		1 mile,
3 miles,	(do. *m.*)		1 league,
60 geographic, or 69½ statute miles,			1 degree,
360 degrees,	(do. *deg.*)		a great circle of the earth.

This measure is used in measuring distances, or any object where length only is taken into consideration.

A *hand* is 4 inches, and is used in measuring the height of horses.

A *fathom* is 6 feet, and is chiefly used in measuring the depth of water, and also cables and ropes.

A *chain* contains 100 links, and is 66 feet, or 4 rods long.

Wine Measure.

2 pints	make	1 quart,
4 quarts,		1 gallon,
42 gallons,		1 tierce,
63 gallons,		1 hogshead,
84 gallons,		1 puncheon,
2 hogsheads		1 pipe, or butt,
2 pipes, or 4 hogsheads,		1 tun.

[illegible] in

[illegible] C.

[illegible] of

[illegible] ple,

[illegible]

Time.

60 seconds,	(marked *sec.*)	make	1 minute,
60 minutes,	(do. *min.*)		1 hour,
24 hours,	(do. *h.*)		1 day,
7 days,	(do. *d.*)		1 week,
4 weeks,	(do. *w.*)		1 month,
52 weeks, 1 day and 6 hours, 12 solar, or calendar, or 13 lunar months,			1 year,

365 days, 5 h. 48 min. 57 sec. & 39 thirds, 1 Julian year.

Thirty days have September, April, June, and November;
February hath 28 alone, and all the rest are thirty-one.
In bissextile, or leap year, February has 29 days.

Circular Motion.

60 seconds,	(marked ″)	make	1 minute,
60 minutes,	(do. ′)		1 degree,
30 degrees,	(do.°)		1 sign,
12 signs, or 360 degrees,	(do. *s.*)		1 great circle of the Zodiac.

Miscellaneous Table.

12 articles of any kind,	make	1 dozen,
12 dozen, (marked *doz.*)		1 gross,
24 sheets of paper,		1 quire,
20 quires,		1 ream,
10 reams,		1 bale,
5 skins of parchment,		1 roll,
7[illegible] words in common law,		1 sheet,
90 words in Chancery,		1 sheet,

Folio is the largest sized book.

Folio	contains	2 leaves,	or 4 pages, in	1 sheet,
Quarto	"	4 do.	or 8 do.	1 sheet,
Octavo	"	8 do.	or 16 do.	1 sheet,
Duodecimo, or 12mo.		12 do.	or 24 do.	1 sheet,
Octodecimo, 18mo.		18 do.	or 36 do.	1 sheet.

Merchantable Grain.

Wheat	60 lb.	to the bushel.
Rye,	58 do.	
Barley,	48 do.	
Oats,	38 do.	

Explanation of Characters commonly used by Arithmeticians, and which are adopted in this work.

= *Equal to*, as 12*d.*=1*s.* viz. 12 pence are equal to 1 shilling.

+ *More*, or sign of Addition; as 4+6=10: viz. 6 and 4 added together, are equal to 10.

— *Less*, or the sign of Subtraction; as 8—5=3; viz. 5 subtracted from 8, leaves, or equals 3.

× *Multiplication*, or sign of Multiplication; as, 6×7=42; viz. 6 multiplied by 7, produce, or equal 42.

÷ *Division*, or the sign of Division; as, 9÷3=3; viz. 9 divided by 3, gives the quotient of, or is equal to 3.

: : Placed in the midst of 4 numbers, denote the numbers to be proportional to each other, and is used in the Rule of Three; thus, as 3 : 6 : : 12 : 24; that is, as, 3 *is to* 6, *so is* 12 *to* 24.

$\sqrt{}$ or $\sqrt[2]{}$ denotes that the square root of the number before which it is placed, is required.

$\sqrt[3]{}$ Denotes that the cubic root of the number is required.

—— Vinculum denotes that all the quantities are considered jointly as one quantity.

Addition and Subtraction Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	5	6	7	8	9	10	11	12	13	14
3	5	6	7	8	9	10	11	12	13	14	15
4	6	7	8	9	10	11	12	13	14	15	16
5	7	8	9	10	11	12	13	14	15	16	17
6	8	9	10	11	12	13	14	15	16	17	18
7	9	10	11	12	13	14	15	16	17	18	19
8	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	16	17	18	19	20	21
10	12	13	14	15	16	17	18	19	20	21	22
11	13	14	15	16	17	18	19	20	21	22	23
12	14	15	16	17	18	19	20	21	22	23	24

To add numbers in this table, look in the left-hand column for one of the numbers given, and in the top line for the other, and in the line of intersection will be found their amount. Ex. add 8 and 5 together. Look in the left-hand column for 8, and in the top line for 5, then at the right hand, at the intersection of the two lines, will be found 13, the amount of 8 and 5.

To subtract, look for the less of the given numbers in the left-hand column, follow its line to the right, until the other number is found; then looking to the top of the line of intersection, and the difference is found. Ex. Subtract 9 from 20. Look for 9 in the left-hand column, and following the line to 20, on the top of the same column stands 11, which is the difference between 9 and 20.

Multiplication and Division Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

For Multiplication; look for the multiplier in the left-hand column, and the multiplicand at the top, and in the angle of intersection of the two lines, will be found the product. Ex. 8×7; look in the left-hand column for 7, and in the top line for 8, and at the intersection of the two lines in which these figures stand, is their product 56.

For Division; viz. to divide 56 by 7: look for the divisor 7 in the left-hand column, and following it to the right, 56 is found; at the top of the table over 56, stands the quotient 8: that is, 7 is contained in 56, 8 times; or, vice versa, 8 is contained in 56 7 times.

Questions relative to the Tables, &c.

1. What are the denominations of Federal money?
2. What are the denominations of American and English money?
3. How is the pence table repeated; how, the shilling table; and how the pound?
4. What are the denominations of Troy weight, and what is its use?
5. What are the denominations of Avoirdupois weight, and what is its use?

6. What are the denominations of Apothecaries' weight, and what is its use?

7. What are the denominations of Cloth measure, and what is its use?

8. What are the denominations of Dry measure, and what is its use?

9. What are the denominations of Long measure, and what is its use?

10. How much is a hand, and what is its use?

11. How much is a fathom, and what is its use?

12. How much is a chain, and what is its use?

13. What are the denominations of land or square measure, and what is its use?

14. What are the denominations of solid or cubic measure, and what is its use?

15. How much does a wine gallon contain; and how much a water gallon?

16. How many solid inches does a bushel contain?

17. What are the denominations of wine measure?

18. What are the denominations of ale and beer measure?

19. What are the denominations of time?

20. How many days are contained in each calendar month?

21. How does a bissextile or leap year differ from other years; and how is it ordinarily found?

22. What are the denominations of circular motion?

23. What are the denominations of the miscellaneous tables?

24. How many lbs. to the bushel, of merchantable wheat, rye, barley, and oats?

25. What characters, or marks, are adopted by arithmeticians, to express equality? Addition? Subtraction? Multiplication? Division? Proportion? to denote the extraction of the Square Root? the Cube Root? or any other root?

ARITHMETIC.

THIS term is derived from the Greek word "arithmos," which signifies number, and, in the plural form, numbers, or respecting numbers. Hence Arithmetic is defined to be, "the art or science of computing by numbers."

It has three fundamental rules, or principles, on which all arithmetical calculations are founded; viz. *Notation*, and *Numeration*, the *Increase*, and *Decrease* of numbers.

Notation and Numeration are, radically, of very different significations; nor have we any single word which expresses the meaning for which both are designed. They are both Latin derivatives; The *former* signifies *to note*, or *mark down;* and its appropriate use is, to mark down in a proper order, the characters or figures of any proposed sum, so as to express the true value intended. The *latter* signifies *to number;* and its use is, after the given figures are thus marked down, to number the figures according to their order in Notation, and then read them, or pronounce their total value. It is, therefore, the union of these two significations, which is implied in this fundamental rule; and this union is more commonly designated by the term of Numeration. Such is its use in the following definition often given, viz.:

"*Numeration* is the art of numbering." It teaches how to express the value of any proposed quantity, by the aid of the following figures; 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The figures from 1, to 9, inclusive, are called *digits*, or *significant figures;* the last is a cipher, which, standing alone, has no value; but when associated with significant figures, enhances their value, by removing them farther from the place of units.

It should be carefully remembered, that figures, or whole numbers, increase in a tenfold ratio, or proportion, from the right hand towards the left. Every removal of a significant figure from the right towards the left, increases the value of that figure tenfold more than it expressed before, and would continue so to do, *ad infinitum*. Were this not the case, there would be no

propriety or foundation for using the terms adopted in Notation, viz. units, tens, hundreds, &c. As it requires ten units, or ones, to make ten, and ten tens to make a hundred, and ten hundred to make a thousand, &c., so the very *terms*, or *names*, made use of in Numeration, are explanatory of their meaning, or signification, and spontaneously follow. But had the ratio of increase been only 5, or 8, the terms of units, tens, &c. would be grossly absurd. The terms then must necessarily have been, units, fives, twenty-fives, a hundred-and-twenty-fives, &c.; or units, eights, sixty-fours, &c., and consequently to have carried according to the ratio of increase, to the next, or left-hand figure. On any other plan, therefore, than that which was devised and adopted, with the greatest ingenuity, of a tenfold ratio of increase, there must have been an insuperable barrier to the ready expression of any considerable number, or quantity, and also in computing the same.

It is furthermore apparent, that other considerations harmonize with the denominations of tens. Any amount whatever cannot only be expressed easily in this tenfold ratio, but it is accomplished solely by the use of only *ten* figures, or characters. These figures, which are expressive of numbers, are borrowed from the Arabics, and are written from right to left, in conformity with the Oriental practice of writing from the right to the left. Hence it is, that our valuation and computation still commence at the right.

Figures are susceptible of two values, viz. *simple* and *local.* They retain a simple value only, when a significant figure stands alone, or occupies the place of units; as 6, which denotes six units, or ones. But the instant the figure is removed from the place of units towards the left, it ceases to retain its *simple* value, and immediately assumes a *local* value. Its local value depends chiefly upon the *place* in which the figure is *located* or placed; or the number of places of its removal from the place of units. Thus 5, in unit's place, would express its *simple* value, or five ones. If removed one place to the left, by annexing a cipher, or any other figure, it now becomes so many *tens*, or ten times its simple value; that is, 5 times ten, making its value 50, and thus giving 5 a *local* value: If two ciphers be

annexed to the 5, it is removed to the *third* place, or place of *hundreds*, and it now becomes so many hundreds, or ten times the value of 50, which makes 500, and thereby increases its *local* value tenfold more than the 50 possessed. Thus it would be constantly increasing in value by every removal towards the left, in a tenfold ratio, agreeable to the terms adopted in Notation, which are exhibited in the following table.

TABLE OF NOTATION AND NUMERATION.

Billions.	C. of M. of Mill.	X. of M. of Millions.	M. of Millions.	C. of Millions.	X. of Millions.	Millions.	C. of Thousands.	X. of Thousands.	Thousands.	Hundreds.	Tens.	Units.
-	-	-	-	-	-	-	-	-	-	-	-	1
-	-	-	-	-	-	-	-	-	-	-	2	2
-	-	-	-	-	-	-	-	-	-	3	3	3
-	-	-	-	-	-	-	-	-	4	4	4	4
-	-	-	-	-	-	-	-	5	5	5	5	5
-	-	-	-	-	-	-	6	6	6	6	6	6
-	-	-	-	-	-	7	7	7	7	7	7	7
-	-	-	-	-	8	8	8	8	8	8	8	8
-	-	-	-	9	9	9	9	9	9	9	9	9
-	-	-	9	8	7	6	5	4	3	2	1	0
-	-	1	2	3	4	5	6	7	8	9	2	3
-	8	7	6	5	4	3	2	1	2	3	4	5
9	9	8	7	6	5	4	3	2	1	5	6	7

One.
Twenty-two.
Three hundred and thirty-three.
4 thousand 4 hundred and forty-four.
55 thousand 5 hundred and fifty-five.
666 thousand 6 hundred and sixty-six.
7 millions 777 thous. 7 hundred and seventy-seven.
88 mill. 888 thous. 8 hundred and eighty-eight.
999 mill. 999 thous. 9 hundred and ninety-nine.
9 thous. 876 mill. 543 thous. 2 hundred and ten.
12 thousands 345 millions 678 thousands 9 hundred and 23.
876 thousand 543 millions 212 thousand 3 hundred and 45.
9 billions 987 thous. 654 millions 321 thous. 5 hund. & 67.

From a strict attention to the above table, two important considerations demand particular care; first, the right placing of the figures, to correspond with the numerical terms for which they are designed; 2dly, the true value of each figure, as it is exhibited in its proper place. Hence the principles of Notation and Numeration should be thoroughly understood, before the learner proceeds to the subsequent rules; as every operation in figures is essentially dependent upon this radical basis.

Although it is seldom that as many as nine places of figures are required to express any given quantity, yet it may be proper to instruct the learner how he can numerate ever so great a number of figures. Next in order to Billions, would be Trillions, Quadrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, &c. &c., following the Latin numerical terms in counting, to a seemingly unlimited extent. It will be remembered, that each of the above terms of Billions, Trillions, &c. would, if the Notation should extend through each term, occupy six places of figures, repeating the same names over and over again, under each respective term, as they were used in their order in the first six figures at the commencement of Numeration, only with this difference; when we arrive at the place of Millions, the right-hand figure of that term, instead of the term units, would be called Millions; and yet it is unit's place, as it respects the term Millions: then, would follow, in regular order, the terms, tens, hundreds, thousands, tens of thousands, hundreds of thousands, prefixed to millions. The same would be true of all the succeeding terms, in the numerations of Billions, Trillions, &c., except the name of the first, or right-hand figure of each respective term in succession. The first figure under each term would take simply the name of the term under consideration, and not that of unit: yet strictly considered, it is the unit's figure of the term to which it is made applicable.

To facilitate the labour, in acquiring a thorough knowledge of Notation and Numeration, it may be useful to separate the given figures, by commas, into parcels of three figures each, beginning at the right hand. The first comma would separate the hundreds from the thousands; the second, the thousands

from the millions; the third, the hundreds of millions from the thousands of millions; and so onward, each comma dividing either the different terms of Notation, or the hundreds from the thousands, of each respective term. A vinculum connecting each respective term will aid in the operations.

&c. &c.	C. of Trillions.	X. of Trillions.	Trillions.	C. of M. of Billions.	X. of M. of Billions.	M. of Billions.	C. of Billions.	X. of Billions.	Billions.	C. of M. of Millions.	X. of M. of Millions.	M. of Millions.	C. of Millions.	X. of Millions.	Millions.	C. of Thousands.	X. of Thousands.	Thousands.	Hundreds.	Tens.	Units.
,	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			3	0	0	0	0	0	3	0	0	0	0	0	3	0	0	3	3	0	3
		2	2	0	0	0	0	2	2	0	0	0	0	2	2	0	2	2	0	2	2
	2	2	5	0	0	0	2	2	5	0	0	0	2	2	5	2	2	5	2	2	5

To assist the learner to express with accuracy any given sum or quantity, agreeably to the design of the preceding illustration, let him place ciphers under the several terms thus respectively connected, separated by commas, as before directed. This being done, place the significant figures of the given sum to be expressed, under the places of their correspondent values; and having thus disposed of all the significant figures, in the given sum or quantity, supply any vacancy of place, or places, with ciphers, should there be any vacancies between these significant figures already arranged.

Example. Express in numbers, 3 Trillions, 3 Billions, 3 Millions, 3 Thousand, 3 Hundred and 3.

Each significant figure should first be expressed separately under its respective name, beginning at the right hand. The last 3 in the given sum is put under unit's place; the second 3, which denotes hundreds, under the place of hundreds; the third 3, in the first place under thousands; the fourth 3, under the first place of millions; the fifth 3, under the first place of billions; and the sixth 3, under the first place of trillions.

Having thus disposed of all the threes, according to their respective values, under their appropriate terms; next supply the vacancies between these significant figures with ciphers, under each intermediate term, so far as the significant figures have extended. Now by enumerating these figures, and carefully observing that each significant figure stands directly under the term it was designed to express, the true value of the given sum will be exhibited. In this way no error is committed; and the intermediate ciphers are indispensably requisite to fill the places between the significant figures, without which their acquired values could not be expressed. Having now numerated them from right to left, to the simple value of each figure, join the name of its proper place, and reading them from left to right, pronounce the sum thus; 3 trillions, 3 billions, 3 millions, 3 thousand, 3 hundred and 3.

Again, to express 22 trillions, 22 billions, 22 millions, 22 thousand and 22, it is necessary to occupy only the two right-hand places respectively, under the several vincula, with the significant figures; and the vacancies between which being supplied with ciphers, the true value is expressed. This will be evident, from numerating the figures, agreeable to their respective collocations, and then reading or pronouncing the sum as expressed.

Suppose the required sum to be, 225 trillions, 225 billions, 225 millions, 225 thousand, 225; then consequently there would be three significant figures placed under the three right-hand figures of each successive vinculum; and the vacancies between these being supplied with ciphers, the precise value would be expressed.

Note. Billions are millions of millions; and Trillions are millions of millions of millions.

It should be recollected also, that the larger the value of one significant figure is over another significant figure, the same number of places being occupied equally by both, the larger significant figure, or figures, will express as much greater value over the smaller significant figure, or figures, as the units in the former exceed the units in the latter.

Thus, 9 trill. 9 bill. 9 mill. 9 thous. 9 hund. and 9 will exceed 1 trill. 1 bill. 1 mill. 1 thous. 1 hund. and 1

by 8 times the whole value of all the smaller figures; or would express 8 times as many units, or ones, in value, as the smaller figures do, although the places occupied by each of the given sums are equal. It is, therefore, obvious, that the larger the significant figures used in numeration, in any given number of places, the greater will be the value expressed; and the smaller the significant figures, expressing a value, the greater will be the units, or ones, between their respective values. Thus, every significant figure up to 9, is composed of so many units or ones, as the figure denotes; and, whether it retains a *simple* or *local* value, the same truth is clearly visible; for 9, 99, or 999, are each composed of so many ones.

These examples are deemed sufficient to ensure accuracy in the learner, relative to the Notation and Numeration of whole numbers, and also how to write and read any proposed sum correctly.

Intimately connected with the Notation and Numeration of whole numbers, is that of Decimals; the latter being predicated on the principle of tens, and derive their name from the Latin numeral "decem," which signifies ten. Decimals are parts of a whole number, or a broken number ingeniously framed on the principle of tens, or a tenfold proportion; but with this difference from whole numbers, viz. whole numbers *increase*, as has already been shown, in a tenfold ratio, from right to left; whereas Decimals, although constructed on the principle of tens, or tenth parts, yet *decrease* in value from the left to the right. Decimals, therefore, are not units, or whole numbers, but expressions of a part of a unit or whole number. They are distinguished from whole numbers, by placing a comma, usually called a *separatrix*, before them: or, when a whole number and a Decimal are united, the separatrix is placed between the whole number and the Decimal. Decimals always suppose the unit or whole number, of which the Decimal is the expression of a part, to be divided into such a number of parts, if they were to be expressed, as would invariably be composed of unity or one, with one or more ciphers annexed to it. Thus the parts of the unit, or whole number, which are usually distinguished by the term *denominator*, would be either 10, 100,

[illegible]

[illegible]

[illegible]

taken out of a hundred; and thus the ratio is decreased tenfold by the farther removal of this figure from the separatrix to the right. If, therefore, the left hand figure of the Decimal were ,4, viz. 4 tenths, by annexing ever so many nines to the right hand of it, the ,4 would never become ,5, or 5 tenths, which would be equal to the half of an unit, or one. Although the annexing of one 9 to the 4 tenths, would make the value 49 hundredths, leaving one hundredth part short of 5 tenths, which is equal to one half of unity; then add a second 9 to the Decimal, which will make 499 thousandths, leaving the deficiency only one thousandth part short of making the ,4 to possess the value of 5 tenths, still it is not 5 tenths, and never could be made such, by the addition, or annexing of ever so many nines, although the addition of each 9, made the deficiency 9 parts out of ten nearer than before the last 9 was annexed. The deficiency would, indeed, be exquisitely small, yet the left-hand figure of the Decimal could never be thus increased to the value of one tenth; or the 4 tenths to the value of 5 tenths. The fact then is clearly discoverable; that numbers may approach nearer to each other for ever, and yet never meet. It is also obvious, that as Decimals *decrease* in value from left to right, they consequently *increase*, in the same ratio as whole numbers, from right to left.

Ciphers prefixed, that is, placed at the left hand of a Decimal, diminish the value of the Decimal in a tenfold ratio, as the significant Decimal is thereby removed one place farther from the separatrix. Thus five, expressed decimally, would be written, 5 tenths, which, being the half of ten, and ten would constitute the whole, or unity, so five tenths would be equivalent to the half of unity, or one. But if a cipher be placed before the five, it would be expressed thus, ,05 hundredths, or only five parts taken out of a hundred parts; or, if two ciphers were prefixed, as, ,005 thousandths, the value would be only five parts out of a thousand parts; because the significant figure in the Decimal, being thus removed farther from the separatrix, by the prefixing of each cipher, is continually decreased tenfold by each removal. But ciphers annexed, or joined to Decimals, do not alter their decimal value. Thus 5, tenths,

,50 hundredths, or ,500 thousandths, are each respectively equal to one half; for 5 is the half of ten, 50 the half of a hundred, and 500 is the half of a thousand. Hence the effect of ciphers with Decimals, is directly the reverse with that of whole numbers.

From this brief survey of the nature of Decimals, it is evident they can be connected with whole numbers, in the operations of Arithmetic, with the greatest facility, by carefully preserving the separatrix in its proper place, so as clearly to distinguish the Decimals from the whole numbers.

The prolixity which has been indulged on this highly important subject of Numeration, justly considered the basis of every arithmetical operation, has arisen from a consciousness that its nature and principles were generally very imperfectly understood by the learner, if understood at all. Students are desirous of acquiring a knowledge of the *whys* and *wherefores* of any branch of science. If the nature and principles of a science are clearly explained, and the rules rendered perspicuous and intelligible by the author, the learner might be enabled to acquire a knowledge of it, with but very little aid from a teacher. His task then becomes pleasant, and his researches are accompanied with profit and delight. But if the explanations and illustrations of the nature and principles are not rendered explicit and luminous by the author, and the task devolves upon the living teacher, all knowledge or assistance might be withheld from the learner. It might happen, in such a case, that the deficiency of the former, and the neglect, or total incompetency of the latter, would leave the student in entire darkness relative to an important branch which he was anxious to investigate and acquire. Hence it is, that a deep-rooted prejudice is frequently excited against the pursuits of a branch of science, or of the arts and sciences generally, arising solely from the sources already suggested, viz. deficiency in defining, or neglect, or incompetency, in instructing. In no branch is it more requisite, that the learner should be well versed, than in a general idea of the nature and principles of Notation. Without this, he is wholly disqualified from entering upon any subsequent rule in Arithmetic, as he could not, in such a situation,

acquire any proper knowledge of the nature, powers, and operations of numbers. With this view of the subject, the learner cannot be too diligent in acquiring an adequate knowledge of this fundamental rule, by repeated and thorough revisions of its nature and principles, and by exercise in many and varied examples, which he can readily supply, before he shall proceed to subsequent rules.

A few examples only will here be given to exercise the learner. Let him write and read the following:

100 millions 55 thousand one hundred and three.

1000 millions ten hundred thousand and twenty.

202 millions 2 thousand 300 hundred, and sixty-seven hundredths decimal.

1 billion 20 millions 20 thousand and twenty, with the Decimals of three hundred and seventy-nine millionths.

One hundred thousand and one hundred, with five tenths, decimal.

Twenty-five thousand and three, with four thousandths decimal.

Seventy-five millions and twenty, with eight thousand six hundred and seventy-seven ten-thousandths, decimal.

Six millions sixty thousand and sixteen, with one millionth, decimal.

Questions relative to Notation and Numeration.

1. How is Aritnmetic defined?
2. Whence is the term derived; and what does it signify?
3. How many and what are its radical rules?
4. What is the first defined to be, or what is it denominated?
5. What is the difference between Notation and Numeration?
6. Why are both words necessary as a technical term for this rule?
7. How is Numeration, in its general acceptation, defined?
8. What does it teach?
9. How many characters or figures are used in computation?
10. What are the first nine figures called; and why?

11. What is the tenth called; and what is its value and use?

12. In what proportion, or ratio, do whole numbers increase; and in what order?

13. What is the effect of this tenfold increase from right to left?

14. What gave rise to the terms adopted in Numeration; and why are not any other terms equally appropriate?

15. What benefits result from this decuple, or tenfold proportion?

16. What other considerations harmonize with the denominations of tens?

17. Whence were modern numbers derived; and why numbered and written from right to left?

18. How could any considerable quantity be expressed, without a tenfold ratio?

19. Of how many values are figures susceptible; and what causes these important distinctions?

20. Why, and how is the local value of a figure justly estimated?

21. What are the terms in Notation, taken in their natural ord·r?

22. How many figures are used under the respective terms of millions, billions, &c.; and what terms are severally prefixed, in regular gradation, to these respective terms; and why?

23. What numbers constitute billions, trillions, &c.?

24. How may any sum be accurately expressed by Notation and Numeration, without liability to mistake?

25. What difference, of actual value, is there between two, or any increased number of significant figures, occupying an equal number of places in Notation?

26. What benefit is derived from a strict observance of the value of which the significant numbers are expressive?

27. What figure, in a given sum, is chiefly expressive of the real value?

28. Whence is the term Decimal derived; or on what principle are Decimals regulated?

29. How do they differ from whole numbers; and by what mark are they distinguished?

30. Into what parts would an unit, or whole number, be divided, of which the Decimal would be the expression of a part. and what would be the appropriate name given to these divisional parts of unity?

31. Is it necessary to express the denominator, to obtain the value of the decimal expression?

32. How is the value of decimal numbers obtained?

33. How are Decimals numerated, and why are the terms used?

34. What is the first, or left-hand figure called; and why?

35. Would the name given to the right-hand figure of the Decimal, in Numeration, express the number of parts of which the denominator would consist, if the denominator were expressed?

36. Which figure of a Decimal expresses its principal value; and how many figures will ordinarily express the value with sufficient accuracy?

37. To what extent would Decimals approximate towards each other without meeting; or when would 5 tenths, by annexing nines to it, be made to equal 6 tenths?

38. What effect is produced to Decimals by annexing ciphers; and why?

39. What is the effect of prefixing ciphers to Decimals; and why?

40. How can whole numbers and Decimals be combined together, and yet accurate answers be obtained?

INCREASE OF NUMBERS.

In all the operations of Arithmetic, numbers are either *increased*, or *decreased*. The Increase of numbers comprises the rules of Addition and Multiplication; the latter of which is only a compendious method of executing the former. As they are equally concerned in the *increase* of quantities, it is expedient to treat of them together, by means of which their

similarity is more readily seen, and their properties more easily understood by the learner.

Their appropriate names will be still retained as before, together with the respective terms peculiar to each, which long-established usages have sanctioned. When taken collectively, they fall under the general name of *Increase;* but referred to individually, they are called by their appropriate names of

ADDITION AND MULTIPLICATION.

Simple Addition teaches to collect several numbers of the *same* denomination into one quantity.

The result of this operation is invariably called *sum total,* or *amount.* These terms are never applicable to any other rule, but belong exclusively to Addition, and should be retained in mind.

RULE.

1. Place the figures of the several given sums respectively under each other, carefully observing to have units stand under units, tens under tens, &c. and draw a line underneath.

2. Commence with the right-hand column, and add the units belonging to it together.

3. Set down under the unit's place the amount, if less than ten.

4. If the amount exceed ten, then set down the right-hand figure of the amount, under unit's place, and add the left-hand figure, or figures to the next column, or row of tens; and thus proceed to the left-

Simple Multiplication teaches to increase, or repeat the greater of two given numbers as often as there are units, or ones in the less number. It hence performs the work of many additions in a summary manner.

The terms belonging to Multiplication are the following:

Multiplicand, the number to be increased, or multiplied.

Multiplier, the number by which the increase is produced, or the multiplying number.

Product, is the number produced by involving or multiplying one number into another; or the result of the multiplication.

The Multiplicand and Multiplier are frequently called *Factors,* or *Terms.*

RULE.

When the multiplier is not over 12, place the multiplier under the multiplicand, so that units stand under units, tens under tens, &c.

Multiply each figure in the multiplicand by the whole multiplier; place the right-hand figure of the product under

hand column, where the whole amount is set down.

Proof. Add the several columns downwards in the same order they were added upwards; if right, the last sum total will equal the first.

Or cut off the upper line, and find the amount of the residue; after which, add the upper line to the last amount, and if this equal the first amount, the work is supposed to be right.

Note. The two proofs already suggested, may not at all times prove the result to be correct. The first, of adding the figures downward, is designed rather as a *revision* of the first addition, by taking the figures in an inverted order, and thereby to prevent the miscalling, or amount of numbers. The same is applicable also to the second rule. Its design is to take the given sums in different orders, so as by revisions, to prevent mistakes.

Proof by the Rejection of Nines.

RULE.

1. Reject the nines in each of the given sums, and place the excesses severally at the right-hand of the given sums, forming one column.

2. Add these several excesses together, reject the nines, and place the excess below.

3. Reject the nines in the sum total, and if this excess equal the last excess, viz. that of remainders of excesses, the work is supposed to be right.

unit's place, and add the left-hand figure, or figures, to the next product arising from multiplying the second figure of the multiplicand. Thus proceed through the multiplicand, and set down all the last product.

PROOF.

Multiply the multiplier by the multiplicand.

Note. The absurd method of proof by Division, when the learner possibly had never heard of the rule, much less of its principles, is now become obsolete. Yet when Division is understood, it affords a substantial rule for the proof of Multiplication.

Proof by rejecting Nines.

1. Reject the nines from both factors, and place the excesses severally at the right hand of each.

2. Multiply these excesses together, and reject the nines from their product.

3. Reject the nines from the total product, and if this excess equal the last excess, the work is supposed to be right.

Addition.		*Multiplication.*	
1.	325	*Multiplic.*	325
	——	*Multiplier,*	5
	325		——
	325		1625 Pro.
	325		——
	325	*Proof.*	
	——	*Multiplier,*	5
Am.	1625	*Multiplic.*	325
	——		——
			1625 Pro.
			——

Note. The illustration of proof by rejecting nines, will be deferred till we come to the *Decrease* of Numbers.

		Proof by nines.	
2.	3476=2	*Mult.*	3476=2
	3476=2	*Mul.*	4=4
	3476=2		
	3476=2	*Pro.*	13904 8
Am.	13904 8		
		Mul.	4
		Mult.	3476
		Pro.	13904

3.	7895	*Mult.*	7895
	7895	*Mul.*	5
	7895		
	7895	*Pro.*	39475
	7895		
Am.	39475		5
			7895
		Pro.	39475

Note. The similarity of the nature of the rules of Increase, is obvious, from the preceding examples. It is observable, that when a column of units, tens, or any other, is added or multiplied together, the right-hand figure of the sum total, or product, is placed directly under the same column, or place: and the left-hand figure, or figures, are carried to the next column, or to the product of the next multiplication as so many tens. Suppose the amount, or product, to be 125: the right-hand figure 5, is placed under the column added, or multiplied; and the two left-hand figures, viz. 12, are added to the next column, or to the product of the next multiplication, as so many tens. Such a method of carrying by tens, is far more easy and rational than that of compelling the learner to anticipate the rule of Division, of which he has no knowledge whatever, and thus to ascertain the number of tens, in the amount, or product. It is hence evident also, that whole numbers bear not only a tenfold ratio to each other, but that it is likewise immaterial whether the tens are obtained by addition or multiplication. If we add

6 sixes together, the amount is 36 : if we multiply 6 by 6, the product is also 36. In either operation there are 3 tens, and 6 left. It is clear then, that multiplication is only a short method of performing several additions; and that each contributes to the increase of numbers, and upon similar principles. More clearly to exhibit the principle of tens, in the rules of Increase, a few examples will be given, in which each sum total, or product, is placed down separately, leaving the tens under their respective and appropriate places, to be included in adding the sum total, or total product.

4. Add
```
723647
723647
723647
723647
------
     28
    16
   24
  12
288
-------
2894588 Sum Total.
```

Multiply
```
723647
     4
------
    28
   16
  24
 12
288
-------
2894588 Total Product.
```

5. Add
```
5678964=0
5678964=0
5678964=0
5678964=0
-------
      16
     24
    36
   32
  28
 24
20
---------
22715856=0 Sum Total, or Amount.
```

Multiply
```
5678964=0
      4=4
--------
      16
     24
    36
   32
  28
 24
20
----------
22715856=0 Product.
```

Proof by nines.

```
6. Add 796897                Multiply 6789768
       543768                               78
       689645                          -------
       ------                               64
           20                              48
          19                              56
         21                             7256
        18                             6442
       21                             5649
      18                             4863
     -------                          56
     2030310 Amount                  49
     -------                        42
                                    ---------
                                    529601904
                                    ---------
```

Note. In the last example in Multiplication, first multiply by 8, and put down the whole product of each respective mul tiplication under its proper figure in the multiplicand. Then multiply by 7, in like manner, observing to place the several products so far below the several figures of the first product, as will leave room for two places of figures. The 7 might have been multiplied first, and the 8 last, and by placing the right-hand figures of the first products under their respective multipliers, the total product would be the same. The nature of carrying by tens is also clearly conspicuous in the foregoing examples.

It is furthermore to be observed, that if the several sums added are of the same value, by putting them down under their respective places, and adding them together, the amount is found. Then, by taking one of these given sums for a multiplicand, and the number of times the given sums are repeated, for a multiplier, the product will be the same as the sum total. In such examples, multiplication is a compendious method of performing the work of many additions. But if the several sums to be added are of different values, the similarity is not so clearly visible, although equally true in fact. In such instances, the several given sums must be individually multiplied by unity, or one; and these several

products added together, will give the product. In such cases, Multiplication would not shorten the work of Addition.

RULE II. *In plultiMication, when the Multiplier consists of several Figures.*

1. Place the multiplier under the multiplicand, units under units, and tens under tens, &c.

2. Multiply each significant figure in the multiplier separately, placing the first figure of each product directly under its multiplier, or the figure multiplied by.

3. Add the several products together in the same order as they stand, and their sum will give the total product.

ADDITION.

7. 25436
86753
54365
67674
Am. up. 234228
208792
Am. 234228

8. 476436
854765
769864
537689
Am. up. 2638754
2638754 *Am downwards.*

9. 1234567
7654321
2345678
8765432
19999998

10. 35465768
86756453
45566767
56667676
224456664

11. 38546589427
96895467896
87659876785
68767658678
56476945542
Amount, 348346538328

MULTIPLICATION.

7. 45736=7 *Mult.*
Proof by nines. 345=3 *Mul.*
228680 21=3
182944
137208
15778920=3 *Pro.*

8. 3547865=2
346=4
21287190 8
14191460
10643595
1227561290=8 *Pro.*

9. 76587456
785
382937280
612699648
536112192
60121152960 *Pro.*

ADDITION.

12.
```
          65735968
          56597576
          89656695
          74888888
          --------
Amount,  286879127
```

13.
```
          75468854
          37856497
            350026
              4545
                87
           1567976
             76354
          --------
Amount,  115324339
```

14.
```
       74307680451
       56791578567
        5876436779
         658765346
          47857658
           3475465
            537846
             64767
              7448
               874
                95
                 9
       -----------
```

15.
```
       62738475865
       39374867578
       56867654765
       47689786487
       45464957564
       70355478679
       -----------
```

MULTIPLICATION.

10.
```
          537265
            3446
      ----------
      1851415190 Pro.
```

```
          2685827
            46879
     ------------
     125908883933 Pro.
```

RULE III.

When there are ciphers on the right hand of either or both the Factors.

Neglect the ciphers, and place the significant figures under each other, and multiply by the significant figures only; and to the right-hand of this product, add as many ciphers as were omitted in both the factors.

11. Multiply 3700 by 5.
```
      5
  -----
  18500 Pro.
```

12.
```
  64360 by 2200
     22
  ------
   12872
  12872
  ---------
  141592000 Pro.
```

13. 675000 × 269000
```
  269
  ------------
  181575000000 Product.
```

ADDITION.

16. 976452
345649
484567
567875

17. 2638765
6473546
7546657
8765498

17.
563054087
308276934
1343276
56879
62734658
147250786

18.
7830065749
296876573
379767
52647654
73346
653
7545465

19.
5407632947
879897463
6756386
748328
5461
84566547
4385772

20.
750734216
78367932
6842795
586374
8675657
287381
73568

MULTIPLICATION.

14. 503400 × 200500
2005

100931700000 *Product.*

15 34870 × 16100
161

561407000 *Product.*

16. 427000 × 92000
92

39284000000 *Product.*

Rule IV.

When the Multiplier is a Composite Number.

A composite number is such as is produced by multiplying two numbers together. Thus, 48 is the composite number of 6 × 8.

Multiply by either of the two numbers first, and that product by the remaining number. The last product will be the total product.

Examples.

17. Mult. 564736 by 48=8×6.
8

4517888
6

27107328 *Product.*

Multiplication.

18. 875684×56=7×8
7

6129788
8

49038304 *Product.*

19. 706434×64=8×8
8

5651472
8

45211776 *Product.*

20. 37546×72=9×8
9

337914
8

2703312 *Product.*

21. 57463756×81=9×9
9×9

4654564236 *Product.*

Rule V.

To multiply by 10, 100, 1000, &c.

Annex to the multiplicand as many ciphers as there are in the multiplier, and the product is obtained.

Multiply 327 *by* 10. *Ans.* 3270. 64720× 10 *Ans.* 647200
539400× 1000 *Ans.* 539400000. 756×100 *Ans.* 75600
12345000× 1000 = 12345000000. 8736× 10 = 87360
750000×10000 = 7500000000.

Questions relative to the Increase of Numbers.

1. What is *Increase*, and of how many parts does it consist?
2. What is Simple Addition?
3. What is Simple Multiplication?
4. What is the result called, where several numbers are collected into one?
5. What are the terms which are applicable to Multiplication; and how is each term defined?
6. What is the first general rule for Addition; and what for Multiplication?
7. How many rules of proof in Addition, and what are they? also in Multiplication, and what are they?
8. How are Addition and Multiplication assimilated? and wherefore carry by tens in both; and why?

9. What is the second general rule for Multiplication? and what is meant by total product?

10. What is the third general rule in Multiplication? and why does it thus produce the total product?

11. What is a composite number, and what is the rule relative to a composite multiplier; and why give the true answer?

12. ow are numbers multiplied by 10, 100, 1000, &c.; and why is the correct answer thus obtained?

COMPOUND INCREASE.

When the numbers to be *increased* are composed of different denominations, but of the same *generic* kind, they belong to Compound Increase; and are divided as in Simple, into Compound Addition and Compound Multiplication. It is called *compound,* because the sums given to be increased are of *different denominations,* as pounds, shillings, pence, and farthings; or tons, hundreds, quarters, &c.; and so of every species or kind relating to time, weight, or measure, which is composed of differen denominations.

By *generic* kind is meant the same species, although it is composed of divers denominations. Thus, pounds, shillings, and pence can be added only with pounds, shillings, and pence; Avoirdupois weight with its own kind; and Troy weight with its own denominations. But pounds, shillings, and pence, cannot be added with Avoirdupois, or any other weight or measure, as they could not be of the same generic kind.

Compound Addition.

RULE.

Federal Money.

1. Place the sums under each other, dollars under dollars, cents under cents, and mills under mills.

2. Begin at the right-hand and add them as in whole numbers, carrying by tens; and taking care to keep the separatrix in a straight line in the

Compound Multiplication.

Multiplication is called compound, when the multiplicand consists of several denominations.

RULE.

Federal Money.

Federal money is multiplied as in whole numbers; and the separatrix is placed as many figures from the right-hand of the product, as there are deci-

Compound Addition.

amount, under the separatrixes used in the several given sums.

$ cts. m.	$ cts. m.
316, 54 0	527, 05 6=7
237, 46 0	275, 62 5=0
134, 29 5	336, 75 0=6
715, 40 7	419, 87 5=7
472, 95 2	178, 37 5=4
647, 37 5	24
$2524, 02 9	1737, 68 1=6
2207, 48 9	1210, 62 5
$2524, 02 9	$1737, 68 1

$ cts. m.	$ cts. m.
129, 5 0	5634, 25 3=1
42, 12 5	7978, 37 6=2
220, 0 0	6856, 62 5=2
167, 75 0	4582, 75 6=1
191, 87 5	6
$750, 80 0	25052, 01 0=6

$ cts. m.	$ cts. m.
630, 50 0	40, 20 0
118, 2 5	115, 18 5
348, 48 0	217, 78 0
413, 34 0	193, 37 5
212, 85 0	203, 50 0
150, 12 5	
$1873, 32 0	$770, 04 0

Compound Multiplication.

mals in both the factors: and if there are not as many places of figures in the product, supply the numbers, by prefixing ciphers.

	$ cts. m.
Multiply	6537,62.5×11
	11
	$71913, 87 5 *Product.*

$ cts. m.
2764, 37 5×12
12
$33172,50 0 *Pro.*

$. cts. $. cts.
367, 66×1, 25
1, 25
183830
73532
36766
$459,57.50 *Product.*

$ ct. m. $ cts.
39, 25.2×1, 75
1, 75
196260
274764
39252
$68,69.100 *Product.*

```
   $ cts. m.      $ cts. m.
   49, 25 0      531,  5 0
   13, 72 0      174, 35 0
  110, 12 5      291, 48 0
   79, 37 5      314, 62 5
   48, 62 5      431, 47 5
   84, 87 5      313, 25 0
 ---------     ----------
 $385, 97 0   $2056, 23 0
```

```
     $     cts. m.
  25727,  32 0
  31195,  49 0
  48421,  72 0
  53678,  37 5
  18436,  12 5
 -------------
 $177459 03 0
```

```
    $ cts. m.     $ cts.
   755, 87 5×3, 37½
     3, 375
   ---------
    3779375
   5291125
  2267625
 2267625
 --------------
 $2551, 07.8125
```

Ans. $2551, 07 8 125/1000 = ⅛

```
 cts. cts.
 ,15×15
 ,15
 -----
 ,0225 = 2¼ Cts.
```

In the above example, the places of figures in the product do not equal the decimal places in both the factors; therefore a cipher is prefixed.

It is plain also, that multiplying decimals together decreases their value; but we shall hereafter find, that division will increase their value.

Note.—If the cents are under ten, a cipher must be placed next to the separatrix.

```
          cts.   cts.
 Multiply 6¼  by 6¼
           ,0625
           ,0625
           -----
            3125
           1250
          3750
 ----------       Mls.
 ,00.390625  Ans. 3.90625
```

Note.—As a dollar is the money unit, and a dime being the tenth part of a dollar, a cent is the tenth part of a dime, or the hundredth part of a dollar, and a mill the tenth part of a cent, or the thousandth part of a dollar; it is evident that any number of dollars, dimes, cents, and mills, is merely the expression of dollars, and the decimal parts of dollars. Thus, 7 dollars, 6 dimes, 5 cents, and 4 mills, are equal to $7, 65*cts.* 4 *m.*, or 7 dollars and $\frac{654}{1000}$, that is, 654 thousandths of a dollar. Although the first right-hand figure, after dollars, may be called dimes, and the second cents, yet the more common usage, in reading the parts of a dollar, is to omit the term dimes, and call the two first figures cents, and the third mills; the latter method might be considered as preferable, as it is sanctioned by general practice, and is so marked by authors in Federal computations. Not only the term dimes, but also that of eagles, is very seldom used.

To add the denominations of money, weight, or measure.

Compound Addition.

RULE.

1. Place the given numbers so that those of the same denomination may stand directly under each other.

2. Begin with the lowest denomination, and add the column together, as in whole numbers. Then ascertain how many of the same denomination of the amount added, make one of the next greater, and how many times it is contained in it; set down the remainder under the column added, and carry as many ones to the next denomination, as there were times contained in the last amount; and thus proceed through all the denomi-

To multiply the denominations of money, weight, or measure.

Compound Multiplication.

RULE.

Place the quantity in the multiplier under the lowest denomination of the multiplicand; and, in multiplying, observe the same rules for carrying from one denomination to another, as in the general rule for Compound Addition.

Note. When accounts are kept either in pounds, shillings, and pence, or in dollars and cents, this method of multiplication is a concise one, to find the value of goods, at so much the yard, gallon, or lb, by only multiplying the given price by the quantity.

In the Compound Increase of Numbers, the proof by re-

Compound Addition.

*Avoirdupois Weight.**

Cwt.	qr.	lb.	oz.
0	3	22	12
	1	19	10
	2	13	9
	3	12	8
2	3	12	7

Apothecaries' Weight.

lb.	℥.	ʒ.	℈.	gr.
5	10	7	2	16
8	11	6	1	12
9	9	5	2	10
7	8	4	1	15
32	5	0	2	13

lb.	℥.	ʒ.	℈.	gr.
1	9	6	2	14
3	11	5	1	11
5	10	7	2	12
4	9	6	1	10
16	6	2	2	7

Cloth Measure.

Yds.	qr.	n.	E.E.	qr.	n.
27	3	3	33	4	3
36	2	1	44	3	3
45	0	3	55	4	2
14	2	2	66	3	3
17	3	1	77	4	2
142	0	2	279	1	1

* Avoirdupois Weight is becoming obsolete.

Compound Multiplication.

Example.

	s.	d.	qr.	
84 yds. at	5	9	1	12×7=84
			12	
£3	9	3	0	*Pro.* of 12 yds.
			7	
£24	4	9	0	*Pro.* of 84 yds.

When no two numbers multiplied together will exactly produce the multiplier, multiply by any two, whose product will come the nearest; then multiply the upper line by what remains, which added to the last product, will give the answer.

What will 53 *yards* amount to, at 15*s.* 6*d.* per yard?

	s.	d.	
	15	6	53=10×5+3
		10	
£7	15	0	*Pro.* of 10 yds.
		5	
38	15	0	*Pro.* of 50 yds.
2	6	6	*do.* of 3 yds.
£41	1	6	*Pro.* of 53 yds.

Compound Addition.

Dry Measure.

Bu	pc.	qt.	pt.
16	3	2	1
12	2	7	0
15	2	6	1
9	3	5	1
11	1	4	0
66	2	1	1

Bu.	pc.	qt.	pt.
24	3	7	1
19	2	4	0
28	3	5	1
46	2	6	1
52	1	2	0
172	2	1	1

Wine Measure.

Gal.	qt.	pt	gi.
27	3	1	3
35	2	0	2
22	3	1	2
43	2	1	0
36	3	1	2
167	0	0	1

Hdd.	gal.	qt.	pt.
34	61	3	1
71	35	2	1
65	43	3	0
56	39	2	1
41	28	1	0
270	20	0	1

Long Measure.

Fur.	po.	yds.	ft.	in.	bc.
0	32	4	2	9	2
	26	3	2	7	1
	37	3	1	8	2
	19	5	2	10	2
	18	4	1	6	0
3	16	0	2	6	1

Lea.	m.	fur.	po.
74	2	5	28
65	1	7	36
48	2	4	18
53	1	6	26
49	2	4	32
2[illegible]	2	5	[illegible]0

Compound Multiplication.

1 Cwt. of feathers, at 4*s.* 4*d.* 2*qrs.* per ℔.

	s.	d.	qrs.	
	4	4	2	12×9+4
		12		
£2	12	6	0	*Product* of 12
		9		
£23	12	6	0	do. of 108
	17	6	0	do. of 4
£24	10	0	0	*Pro. of* 112 ℔s.

505 yards of cambric, at 12*s.* 7*d.* 2*qrs.* per yard.

	s.	d.	qrs.	
	12	7	2	10×10×5+5
		10		
£6	6	3	0	
		10		
63	2	6	0	
		5		
315	12	6	0	*Pro. of* 500
3	3	1	2	
£318	15	7	2	*Pro. of* 505.

To find the value of a Hundred-weight, by having the price of one pound.

If the price be farthings, multiply 2*s.* 4*d.* by the farthings in the price of one ℔. Or, if the price be pence, multiply 9*s.* 4*d.* by the pence in the price of one ℔, and in

Compound Addition.

Land, or Square Measure.

Acrs.	*roods.*	*rds.*	*Sqyds.*	*sqft.*	*sq in.*
463	3	33	0	8	136
528	2	29		7	124
371	2	22		5	121
436	2	16		4	106
282	2	12		6	92
2083	1	32	3	7	3

Time.

Years.	*m.*	*w.*	*d.*	*h.*	*min.*	"
1776	7	3	6	22	42	38
2014	9	2	4	18	38	46
1062	8	3	5	12	44	24
831	7	2	3	15	52	49
725	6	3	4	13	27	31
6411	2	0	4	11	26	8

Note. 13 Weekly or lunar months make one year; and 12 solar months, as January, February, &c. make a year.

It is said that 13 months, 1 day, and 6 hours constitute a Julian year. This is called *Julian*, in honour of Julius Cæsar, who caused the time to be corrected.

52 weeks, composed of 7 days each, would make but 364 days; it would therefore require 1 day and 6 hours to be added, to make a year, or 365 days, 6 hours. But, strictly speaking, a year is 365 days, 5 hours, 48 minutes, 57 seconds, and 39 thirds.

Compound Multiplication.

either case the product will be the answer.

Two shillings and 4 pence is one farthing on every 112 ℔s. or Cwt. Thus 4 in 112 is 28 pence; and 28 pence is 2 shillings and 4 pence. This is multiplied by the number of farthings in the price of one ℔.

What will 1 Cwt. of hides come to, at 2*d.* 1*qr.* per ℔.?

	s.	*d.*	
	2	4	farthings in Cwt.
2¼*d.*=		9	do. in a ℔.
£1	1	0	price of Cwt.

What will 1 Cwt. amount to, at 2¾*d.* per ℔.?

	s.	*d.*	
	2	4	farthings in 1 Cwt.
		11	farthings in 2¾*d.*
£1	5	8	price of 1 Cwt.

Nine shillings and 4 pence is one penny a ℔. on a Cwt., because 9 times 12 is 108, which is 9 shillings, and 4 pence are left in the pence to make 112.

What will 1 Cwt. of cheese come to at 7*d.* a ℔.?

	s.	*d.*	
	9	4	=112*d.*
		7	the pence in 1℔.
£3	5	4	value of 1 Cwt.

What will 1 Cwt. of butter come to, at 11*d.* per ℔.?

	s.	*d.*	
	9	4	
		11	
£5	2	8	Amount of 1 Cwt.

Compound Addition.

Circular Motion.

S.	°	′	″
8	22	45	48
10	21	55	56
9	15	42	34
7	20	33	28
36	20	57	46

S.	°	′	″
11	28	52	59
9	22	48	42
1	12	58	46
2	18	34	38
25	23	15	5

Solid, or Cubic Measure.

Tons.	ft.	in.
216	26	1262
321	37	1112
296	35	276
364	24	653
1200	3	1575

Cords.	ft.
65	124
72	87
54	113
48	25
241	93

Note. 40 Feet of round timber, or 50 of hewn, make a ton.

How many days are there in bissextile, or leap year, when February has 29 days, reckoning according to the following data, viz.

30 Days have September, April, June, and November; February hath 28 alone, (except leap year); and all the rest have 31? *Ans.* 366.

How many oranges would a boy have for his wages, who earned 30 in January, 60 in February, 120 in March, and so on, doubling the last quantity every successive month through the year? *Ans.* 245700.

Compound Multiplication.

Note.—If the number of ℔s. be 60, multiply 1*s.* 3*d.* by the farthings in a ℔.; or, if the price be pence, multiply 5*s.* by the pence, &c. &c.

The pedlers' rule, so called, by the nett hundred, or 100℔s. may sometimes be convenient.

RULE. Bring the pence and farthings, of the price of one ℔., all into farthings. Then double these farthings, and call the amount so many shillings; after which, add to the shillings as many farthings as are in the given price for pence, and the sum total is obtained.

What will 100℔s. of flour come to, at 2¼*d.* per ℔.?

2¼*d.* = 9 farthings.
2 double them,

18 they become *s.*
9 called *d.* added.

£0 18 9 *Amount.*

100×9 the price.
9

4|900 farthings

12|225 pence

18*s.* 9*d.*

Or, 100×2¼÷12*d.*
2

200
25

12|225

18*s.* 9*d.*

Compound Addition.

How much is 12 times 2*s.* 6*d.*? *Ans.* £1 10*s.*

Practical Questions in the Rules of Increase, either Simple or Compound.

If John give me 67 oranges, and Peter 87, and George 55, how many should I have? *Ans.* 209.

David was born in the year 1808; when will he be 22 years old? *Ans.* 1830.

A merchant finds himself indebted thus; to *A* 25*l.* 16*s.* 4*d.*; to *B* 42*l.* 12*s.* 2*d.*; to *C* 20*l.* 10*s.* 4*d.*; to *D* 16*l.* 5*s.* 9*d.*; to *E* 12*l.* 18*s.* 7*d.* Pray how much did he owe? *Ans.* £118 3*s.* 2*d.*

A merchant purchased from another 80 barrels of flour for $5, 50 per barrel; of another 110 barrels, at $5, 75; from another 200 barrels, at $6, 62½. How many barrels did he purchase, and how much did the flower cost him?

Ans. 390 bls. and paid $2197, 50.

A merchant purchased 22 pieces of broad cloth: each piece contained 19 yards at 3*s.* 4*d.* What quantity did he purchase? *Ans.* 434½ *yds.*

A person travels 36½ miles a day during 27 successive days. What distance did he travel? *Ans.* 985½ *Miles.*

A grocer bought 8 hogsheads of sugar, each weighing on an average 12*cwt.* 1. 14*lbs.* What was the weight of the whole? *Ans.* 99 *Cwt.*

Compound Multiplication.

What will 100 come to, at 2½*d.*?

2½*d.*=10 farthings
2
20= shillings
£1 0 10

What will 100 come to, at 2¾*d.*?

2¾*d.*=11 farthings
2
£1 2 11

What will 100 come to, at 3*d.*?

3*d.*=12 farthings
2
1 4 0
12*d.*= 1
£1 5 0

3*d.*=¼ shilling. 25=¼ hund.

What will 100 come to, at 4*d.*?

4*d.*=16 farthings
2
32 shillings
16*d.*= 1 4
£1 13 4

4*d.* is ⅓ of a shilling, 33*s.* 4*d.* is ⅓ of a hundred.

Compound Addition.

A silversmith purchased 12 ingots of silver, each averaging 3lbs. 6*oz.* 16 *pwt.* and 20*grs*., what was the weight of all?

Ans. 42lbs. 10*oz.* 2*pwt.*

A farmer sold 276 bags of wheat, each containing 2 *bush.* 1 *pc.*, at $1,25 per bushel. How many bushels did he sell; and what did it come to?

Ans. 621 *bush.* cost $776,25.

A grocer sold 8 chests of tea, the average weight of each was 2 *Cwt.* 2 *qrs.* 8 lbs. at 90 cents the lb. What quantity of tea did he sell, and how much did he receive?

Ans. 20 *Cwt.* 2*qrs.* 8lbs.
Received $2073,60.

A teacher had 3 apartments in his building. In one were 12 linguists, at 12 dollars per quarter; in another 27, at 8 dollars; in the third, 31, at 6 dollars 50 cents, *per quarter*. How many pupils had he, and what was his quarterly income? *Ans.* 70 pupils, and Rec. $561,50.

How much wine in 12 casks, each containing 63 *gal.* 2 *qts.* 1 *pt.;* and what would it come to, at $1, 87½ the gallon?

Ans. 763 *gals.* 2 *qts.*
Amount $1431, 56.2½.

In 9 parcels of wood, each containing 4 *cords*, 76 *feet*, at $4, 25 the cord. How much wood, and what would it amount to?—*Ans.* 41 *cds.* 44 *ft.*
Amount $175, 71.

In 7 fields, each containing 97 *acres*, 3 *roods*, and 30 *rods;* how much land?

Ans. 685 *acrs.* 2 *rds.* 10 *rds.*

Compound Multiplication.

To multiply Numbers, whether whole or decimal, so that the several Products will incline to the right-hand.

Rule.—Place the multiplier under the multiplicand, so that either tens, hundreds, or thousands in the multiplier may stand under the unit's place of the multiplicand. Begin with the figure in the multiplier, which stands under the units of the multiplicand, and place the first figure of the product directly under it: if figures stand at the left of the figure already multiplied, its product will be removed one place to the left, and the product of the several right-hand figures will stand respectively under the figure which is multiplied. At all times, the right-hand figure of a product is placed under the figure multiplied, whatever may be the position of the multiplier, in relation to the multiplicand.

Multiply 8253 by 826.

```
   826 Multiplier.
 -------
 66024
  16506
   49518
 -------
 6816978 Product.
 -------
```

Multiply 9876 by 9405.

```
      9405
  ---------
  88884
   395040
    49380
  ---------
  92883780 Pro.
  ---------
```

Compound Addition.

A person is possessed of 2 dozen of silver spoons, each weighing 3 *oz.* 16 *pwt.* 12 *grs.*; 1½ dozen weighing 2 *oz.* 18 *pwt.* and 9 *grs.*; and 2 silver tankards, weighing 22 *oz.* 10 *pwt.* and 8 *grs.* What is the weight of the whole?

Ans. 15 *lb.* 9 *oz.* 7 *pwt.* 10 *grs.*

A person sold in market, 57 chickens, at 20 *cts.* each; 49 turkeys, at 92 *cts.*; 72 geese, at 37½ *cts.*; and 25 partriges, at 30 *cts.* each. How many in the whole, and what did they amount to?

Ans. 203 fowls; amt. $90,98.

If 8 boys have each 12 cats, and each cat 4 kittens; what is the whole number of their stock?

Ans. 480.

Suppose a farmer's stock consisted of 8 horses, that devoured each 2½ tons of hay; 12 oxen, 2 tons, each; 20 cows, 1½ tons, each. What is the number of his stock, and how much hay would they consume in one winter?

Ans. 40 stock; hay 74 tons.

If a man drive to market 50 fat oxen, valued at 75 dollars, each; 40 cows, at 25 dollars each; 58 calves, at 5½ dollars each; and 60 sheep, at $1,25 each; what is the whole number of the drove, and how much would they amount to?

Ans. 208 stock, amount $5144.

If a school, consisting of 45 boys, had each 3 Latin books, 3 Greek, 3 French, and 6 English; what number of books in the whole? *Ans.* 675.

Compound Multiplication.

Multiply 261986 by 7638.

```
      7638
    -------
   1571916
  1833902
    785958
    2095888
  ----------
  2001049068 Product.
```

Multiply 23,476 by 7,35.

```
      7,35
    ------
164,332
   7,0428
   1,17380
  --------
 172,54860    Product.
```

In multiplying decimals from left to right, the first figure of each product is placed one remove to the right successively, and then the separatrixes are kept in a straight line.

Multiply $26,3754 by 22,6532.

```
        22,6532
       --------
  52,7508
 527,508
  15,82524
   1,318770
       791262
        527508
     ----------
Pro.$597,48721128
```

Compound Multiplication.

Multiply	675432 by 56789.
	56789
	3377160
	6078888
	4052592
	4728024
	5403456
Pro.	38357107848

Note.—This last example shows the absolute *necessity* of placing the right-hand figure of the first product directly under that figure in the multiplier which is involved for the time being. No matter which figure in the multiplier is taken first, second, or third, in order. The result will be the same.

Questions relative to Compound Increase.

1. What is Compound Increase?

2. What is meant by *generic kind*; and why not mingle cwts., dollars, and acres?

3. What are Compound Addition and Multiplication?

4. What are the denominations of Federal money, and their relative values?

5. How is Federal money added and multiplied?

6. How is the amount, or sum total, pointed off in Federal money? how in Multiplication? and how in case of deficiency of places of figures in the product?

7. How are cents expressed when under ten?

8. Why carry by tens in Federal money?

9. Why carry by different numbers in the various tables of weights and measures? and why by different numbers usually in the same table?

10. What are the rules for Compound Addition and Multiplication?

11. When the multiplier is a composite number, what is the rule?

12. When the multiplier is not a composite number, how get the answer?

13. What constitutes an exact Julian year?

14. How is the value of a Cwt. found, when the price of one lb. is given either in pence or farthings?

15. How is the value of 60℔s. found, when the price of one ℔ is 1*s.* 3*d.* or 5*s.*?

16. How does the pedlers' rule furnish the true value, by the nett hundred?

17. How may numbers be multiplied, so, that the several products shall incline to the right?

18. How multiply numbers, so that the several products shall alternately incline to the left or right; or in any prescribed form?

19. To which of the rules do the appropriate terms belong, and what is their import severally, viz. Amount, Product, Sum Total, Factors, or Terms, Sum, Multiplicand, Multiplier, and Total Product? and where are their several positions?

DECREASE OF NUMBERS.

The *Decrease* of Numbers is the opposite of *Increase.* It has been already remarked, that in every operation of figures, numbers were necessarily increased, or diminished. The increase, to which we have already been attending, embraced the rules of Addition and Multiplication. Decrease, which diminishes a quantity, is composed also of two rules, viz. Subtraction and Division. When taken collectively, they retain the general term of *Decrease;* but when referred to individually, they assume their old appropriate names of Subtraction and Division. The nature and operations of each, and the similarity of the principles on which they are founded, will now demand an attentive investigation.

SUBTRACTION.

Simple Subtraction teaches to find the excess, or difference, between any two given numbers of the same denomination. The greater of the two numbers is called the *Minuend.*

DIVISION.

Simple Division teaches to find how many times one whole number is contained in another; and also what remains, if one number does not exactly measure the other. It is a *concise*

SUBTRACTION.

from minuo, which signifies to *diminish*, or to be *lessened*. The less number is called the *Subtrahend*, from subtraho, to *draw out*, or take from. The result of the operation is called, *difference*: (not *remainder*, for this term belongs exclusively to Division.)

RULE I.

1. Place the less number under the greater, with units under units, tens under tens, &c., and draw a line under them.

2. Begin at the right-hand, and take the lower figure from the one above it, and set down the difference.

3. If the figure in the lower line is greater than the one above it, add ten to the upper figure; from which number so increased, take the lower, and set down the difference, carrying one to the next lower number; and thus proceed as before, until the whole is finished.

PROOF.

Add the difference to the subtrahend, or less sum, and if the amount equal the greater, or minuend, the work is supposed to be right: or, if the difference is subtracted from the minuend, and it leaves a difference equal to the subtrahend, it is right.

DIVISION.

way of performing several Subtractions.

There are four appropriate terms used in Division.

1 The *Dividend*, or number given to be divided.

2. The *Divisor*, or number given to divide by.

3. The *Quotient*, or answer to the question, which shows how many times the divisor is contained in the dividend.

4. The *Remainder*, which is always *less* than the divisor, and of the same *name*, or kind, with the dividend.

RULE I.

1. Place the divisor at the left-hand of the dividend.

2. Consider how many times the divisor is contained in so many of the left-hand figures of the dividend, as are necessary to contain the divisor; and place the number sought, at the right-hand of the dividend, for the first figure in the quotient.

3. Multiply the divisor by this quotient figure, and place the product under the left-hand figures of the dividend already used.

4. Subtract this product from the dividend, and call the difference the first remainder.

5. To the right-hand of this remainder bring down the next figure in the dividend.

6. Consider how many times the divisor is contained in this number, place the figure at the right-hand in the quotient;

SUBTRACTION	DIVISION.
Also, by rejecting 9's, proceed as in Addition, only subtract the excesses of the minuend and subtrahend, instead of adding.	then multiply, subtract, bring down, and divide, until all the figures in the dividend are brought down. The quotient is the answer.

Note, explanatory of proof by Nines.

If several sums be given to add together; these individual sums may all be divided by any divisor, and each quotient is noted down. If any remainder, after the first quotient, add this remainder to the next sum to be divided; again, divide, note the quotient, and add the remainder, if any, to the next sum. Proceed thus through all the divisions, and note the *last* remainder. Find the sum total of the several given sums, already divided; and also the sum of the several quotients which have been obtained. Divide the amount of the given sums by the same divisor as before, and the quotient thus given, will equal the sum of the quotients already found; and the *remainder* will be the same as the last remainder

	Sms.	*Quot.*	*Rem.*	
Ex.	378 ÷ by 8 =	47.———	2.	This added to 437 + 2 = 439.
	437 ÷ by 8 =	54.———	7.	This added to 268 + 7 = 275.
	268 ÷ by 8 =	34.———	3.	*Last Remainder.*
	8|1083	135 *tot. quot.*	3	*Remainder.*
	135		3	*Remainder.*

It is no matter what divisor is used; the result will be similar. From this similarity of the remainders, is obviously inferred the proof of rejecting the 9's.

If a cipher, or ciphers, be annexed to any of the digits, and the sum divided by 9, the quotient will be the same as the digit to which is annexed the cipher or ciphers, and also the remainder will be the same as the digit, or quotient.

Divide all by Nine.

Ex. 9	)20	30	40	50	60	70	80	90
	2,2	3,3	4,4	5,5	6,6	7,7	8,8	9,9

Divide all by Nine.

Ex. 9	)200	300	400	500	600	700	800	900	90000
	22,2	33,3	44,4	55,5	66,6	77,7	88,8	99,9	9999,9

It is apparent that any digit, with a cipher annexed, contains so many tens, as the significant figure denotes ones; it must therefore contain an equal number of 9's, and also a remainder of an equal number of units.

Hence, if any sum be divided by 9; the *remainder*, after this division, would always equal the *remainder*, arising from having added the individual figures together, which constituted the sums already divided, then dividing the same by 9.

Ex. Divide by 9, the following sum: viz.

9) 5765 (640,5 *Remainder.*
5+7+6+5=23÷9=2,5 *Remainder.*

Also divide by 9 the following sum: viz.

9) 65432 (7270,2 *Remainder.*
6+5+4+3+2=20÷9

2,2 *Remainder.*

SUBTRACTION.

Examples to illustrate each Rule; and by which their similarity is exhibited.

Subtraction.

1. From 4729 *Minuend,*
Take 618 *Subtrahend,*

4111 *Difference.*

Proof, 4729 *Diff. & Subt.*

618 *Subtrahend.*

Divison.

RULE I.

Div. Divid.

1. 25|625|25 *Quotient.*
50

125
125

2. From 67528 *Minuend,*
Take 42635 *Subtrahend,*

Difference, 24893 *Sub. from Min.*

42635 *Subtrahend.*

2. Divide 6072 by 276.

Div. Divid.
276|6072|22 *Quotient.*
552

552
552

DIVISION.

Proof.

Multiply the divisor and quotient together; and if there be any remainder, add it to the product, which will then equal the dividend. Or, by adding the remainder and subtrahends together in regular order, will give the dividend.

A Third Method of Proof by Excess of Nines.

1. Reject the nines from the divisor, and also from the quotient.

2. Multiply these two excesses together, and to their product add the remainder, *if* any. Then reject the nines from this amount, and note the excess.

3. Reject the nines from the dividend; and, if this excess equal the last excess, the proof is deemed correct.

Note.—The excesses in the divisor and quotient are *multiplied,* and the remainder added, because this is necessary, as in the first proof, to equal the dividend.

RULE II.

When the Divisor does not exceed 12.

1. Place the divisor on the left-hand of the dividend.

2. See how often the divisor is contained in the first, or left-hand figure, or figures of the dividend; *under* the dividend already used, place the quo-

SUBTRACTION.

Proof by Nines.

3. From 85293=0
Take 56729=2
Difference, 28564=7

3. Divide 4225 by 325.

325|4225|13=4
325 1
975 4 *Div. and Quot.*
975
4 *Dividend.*

4. From 95763=3
Take 67892=5
Difference, 27871=7

5. From 1056
Take 176 1st.
880
176 2d.
704
176 3d.
528
176 4th.
352
176 5th.
176
176 6th.

DIVISION.

tient figure. Subtract the product of this quotient into the divisor, from the dividend used, *mentally*, and consider the remainder, if any, as placed before the next figure in the dividend.

3. See how many times the divisor is contained in this new dividend; and when found, place it under this dividend; then multiply, subtract, &c., until all the figures in the dividend are used.

Proof.

Similar to the first proof under rule first.

Rule III.

1. When there are ciphers at the right-hand of a divisor, they may be cut off; also cut off the same number of figures from the right-hand of the dividend; and then proceed as before.

2. Remember that the figures cut off from the right-hand of the dividend, must be placed at the right-hand of the last remainder, and thus become a remainder to the whole divisor.

Rule IV.

1. When the divisor is a composite number, find the two numbers which produce it.

2. Divide the given dividend by one of those figures; and that quotient being divided by

SUBTRACTION.

Division.

Divide 1056 by 176.

176)1056(6
1056

Proof by Nines.

Divisor, 176=5
Quotient, 6=6

30=3

Excess in Dividend, 3

Note.—In the last example subtracted and divided, the number of subtractions, viz. 6, is equal to the quotient arising from the division of the minuend by the subtrahend. Hence one division accomplishes 6 subtractions; and both diminish in quantity.

Subtraction.

6. From 3744=0
Take 234=0

Difference 3510 0

Division.

3744÷234

234)3744(16
234

1404
1404

Were the subtractions continued 16 times, there would be

DIVISION.

the other; the last quotient will be the answer.

3. Find the total remainder by multiplying the last remainder by the first divisor, and adding to it the first remainder.

RULE V.

To divide by 10, 100, 1000, &c.

Cut off as many figures from the right-hand of the dividend as there are ciphers in the divisor; and the figures so cut off are the remainder, and the other figures of the dividend are the quotient.

Subtraction.

From 6733
Take 1675

5058 1st. *Diff.*
1675

3383 2d.
1675

1708 3d.
1675

33 4th.

Division.

1675)6733(4
6700

33 *Rem.*

1675×4+33=6733 *Proof*

SUBTRACTION.

no difference as there is no remainder in division. If division should leave a remainder, the quotient would show the number of subtractions, after which, the same difference would remain.

Thus:

From 632
Take 125
———
507 1st. *Diff.*
125
———
382 2d.
125
———
257 3d.
125
———
132 4th.
125
———
Diff. 7 5th.
———

Division.

632÷125

125)632(5
625
———
Rem. 7
———

Proof by Nines.

Divisor, 8
Quotient, 5×8+7
47—2
Dividend—2

DIVISION.

Divide 7516 by 16.

16)7516(469
64 16
——— ———
111 2826
96 469
——— ———
156 7516 *Proof.*
144 ———
———
12
———

25)5677(227
50
———
67
50
———
177
175
———
2
———

35) 13748 (392
105 35
——— ———
324 1988
315 1176
——— ———
98 13748 *Proof.*
70 ———
———
28 *Remainder.*
———

525)65478(124
525
———
1297
1050
———
2478
2100
———
378
———

SUBTRACTION.

From 2274
Take 563

1711 1st. *Diff.*
563

1148 2d.
563

585 3d.
563

Diff. 22 4th.

Division.

563)2274(4
2252

Rem. 22

Proof by Multiplication.

563×4+22
4

2274

Subtraction.

From 18295
Take 4567

13728 1st. *Diff*
4567

9161 2d.
4567

4594 3d.
4567

27 4th.

DIVISION.

Note. As multiplication performs the work of many additions, so Division accomplishes that of many Subtractions. They are respectively contractions of the simple rules; and, when compared together, their effects are entirely opposite. As the multiplier always shows how many additions would give the required number so the quotient in Division shows how many subtractions are required to exhaust the dividend. It follows then, that if the product were divided by the multiplicand, the quotient *would* be the multiplier; or, if the product were divided by the multiplier, it would give the multiplicand.

Also, in Division, divide the dividend (after subtracting the remainder, if any,) by the quotient, and it will give the divisor.

SUBTRACTION.

4567)18295(4	657403754
18268	428534765
27 *Remainder.*	*Diff.* 228868989

4567×4+27
4

18295 *Proof.*

From	6750	From	47639
Take	5769	Take	26786
Diff.	981	*Diff.*	20853

From	345678	76543285
Take	134789	35987456
Diff.	210889	40555829

	578654	73366=7
	479867	59673=3
Diff.	98787	13693=4
Min.	578654	
Subt.	479867	

7059643	2345678	
2546857	1456789	
4512786	888889	*Diff.*
7059643	1456789	*Subt.*
	2345678	*Min.*

Note. When the lower figure is greater than the upper, ten is borrowed, and put with the upper figure, and it is afterwards repaid, by adding one to the next left-hand figure of the subtrahend. The reason is obvious, that if two numbers are equally increased, their difference will not be altered. If we borrow ten, to add to the figure in the minuend, we also pay one to the next higher place of the subtrahend, which is equal to ten. Should it be asked, how borrowing from the minuend, and paying to the subtrahend, can cancel the loan? it is answered, the subtrahend is thereby increased, so as to make an equal balance with the minuend for the favour conferred.

Compound Subtraction.

Liquid Measure.

Tuns.	hhd.	gal.	qt.
43	2	45	2
38	3	59	3
4	2	48	3

Gal.	qt.	pt.
78	1	1
69	2	1
8	3	0

Dry Measure.

	Bu.	pc.	qt.
	102	2	2
	100	3	3
Diff.	1	2	7

Pc.	qt.	pt.
4	5	0
3	6	1
0	6	1

Time.

	Years.	m.	w.	d.	h.
	74	5	3	5	15
	72	6	2	6	18
Diff.	1	12	0	5	21

	D.	h.	min.	sec.
	95	12	30	45
	87	16	40	50
Diff.	7	19	49	55

Motion.

	S.	°	′	″
	10	28	42	52
	7	29	52	55
Diff.	2	28	49	57
Proof	10	28	42	52

Compound Division.

	£.	s.	d.	qr.	
Divide by 7)	236	7	7	2	
£	33	15	4	2	*Quo.*
				7	
£	236	7	7	2	*Prf.*

£.
5 left.
20

7)107 7 added.

15 2 left.
12

7)31 7 added.

4 3 left.
4

7)14 2 added.

2

	£.	s.	d.	qr.
Divide by 12)	538	12	8	0
£	44	17	8	$2\frac{2}{3}$
				12
£	538	12	8	0

	£.	s.	d.	
Divide by 9)	137	9	6	
£	15	5	6	*Quo.*

Compound Subtraction.

Motion.

	S.	°	′	″
	7	25	18	56
	5	26	53	28
Diff.	1	28	25	28
Proof,	7	25	18	56

Practical Questions in Simple and Compound Subtraction and Division.

From 56 thousand and 97, take 2 thousand 3 hundred and 27. *Ans.* 49770.

From 2 millions take 9 hundred and 99 thousand. *Ans.* 1001000.

From 877 millions, take 977 thousand. *Ans.* 876023000.

From 1 million of dollars, take 3 cents. *Ans.* $999999,97.

What number taken for these five numbers, viz.—25+32+58+29+39, will leave a hundred and twenty-five? *Ans.* 52.

A drover purchased 376 oxen for $22560; and sold 222 for $15540; how many oxen had he left, and what did they stand him in? how much did he pay a head for them, when he purchased? how much did he receive for those sold by the head? and what do the remainder stand him in by the head?

Ans. 154 left, stand him in $7020; gave $60 a head—sold for $70; those left stand him in $45, 58 4½.

Compound Division.

When the divisor is a composite number, divide by one of those numbers first; and the quotient by the other number.

	£.	s.	d.	
Divide 3)	30	15	6	by 21=3×7
7)	10	5	2	
£	1	9	$3\frac{5}{7}=\frac{15}{21}$	*Ans.*

	£.	s.	d.	q.	
Div. 8)	45	16	0	0	by 32=8×4
4)	5	14	6	0	
£	1	8	7	2	*Ans.*

	£.	s.	d.	q.	
Div. 6)	146	12	10	0	by 42=6×7
7)	24	8	9	$2\frac{1}{6}$	
£	3	9	9	$3\frac{5}{7}=\frac{34}{42}$	

	£.	s.	d.	qr.	
Div. 8)	388	4	8	0	by 64=8×8
8)	48	10	7	0	
	6	1	3	$3\frac{4}{8}$	

Compound Subtraction.

A grocer bought 25 *cwt.* 3 *qrs.* 14 *lbs.* of sugar, for $207; and sold 18 *cwt.* 1 *qr.* 14 *lbs.* for $183, 75; what was there left, and how much did it stand him in? *Ans.* 7 *cwt.* 2 *qrs.* left, It stood him in $23, 25.

The war between England and America, commenced April 19th, 1775; and peace took place January 20th, 1783. How long did the war continue?

	Y.	*m.*	*d.*
	1783	1	20
	1775	4	19
Ans. Yrs.	7	9	1

What is the difference between twice three and sixty, and twice sixty-three? *Ans.* 60.

Methuselah was 969 years old when he died; how old was he 777 years before his death? *Ans.* 192.

A man is indebted to *A* $45; to *B* $39, 50; to *C* $125, 75; to *D* $97; to *E* $65, 62,5; to *F* $163; he has in ready cash $1500. How will his funds stand, when he has cancelled the above debts?

Ans. $963, 62,5 left.

A person, at his decease, left property to the amount of $12,000. He directed by his will $540 to be given to the poor; his widow to receive $3,000; and the residue equally divided among his five children, What had each child? *Ans.* $1692 each.

Compound Division.

From a piece of cloth of 114 yards, directions were given to make 12 coats from one quarter of the above piece. How much cloth did each coat contain? *Ans.* 2*yds.* 1*qr.* 2*n.*

If 11 tons of hay cost £25 16*s.* 6½*d.*, how much is it per ton? *Ans.* £2 6*s.* 11½*d.*

If 20 gallons of brandy cost £10 6*s.* 8*d.*, what is that per gallon? *Ans.* 10*s.* 4*d.*

If 1378lbs. of cheese cost £35 17*s.* 8*d.* 2*qrs.*, how much is it per lb.? *Ans.* 6*d.* 1*qr*

A party of 11 persons expended at one time £4 2*s.* 11 *d.*; how much would be an equal individual share?

Ans. 7*s.* 6½*d.*

A person receives a yearly income of $750; and his daily expenses during the year are on an average 56 cents. How much has he saved at the close of the year? *Ans.* 545,60.

What must a person's income be, whose expenses are $3,50 by the week, that he may be enabled to lay up at the year's end, $450?

Ans. $632.

A teacher had 50 pupils; and it was stipulated, that their tuition should be 8 dollars per quarter, in case they were industrious in their studies. If not industrious, such should pay only 5 dollars. It proved that 30 were industrious, and 20 were lazy. How much loss did the teacher sustain by lazy boys? *Ans.* $60.

N. B. And the boys much more.

Compound Subtraction.

If a penny is taken from £1000, then one shilling added; one shilling and eleven pence again taken from it; what sum remains?

Ans. £999 19.

A farmer had four farms; the first contained 167 *acres*, 2 *roods*, and 20 *rods;* the second 215 *acres*, 1 *rood*, 15 *rods;* the third 192¼ *acres*, the fourth 187½ *acres*. These were to be divided equally between his four sons. How many acres did each son have?

Ans. A 190 2 28¾.

Which is the greatest, 15 times 50; or 11 times 60; and what is the difference?

Ans. the 1st, greatest; the difference 90.

Questions relative to Compound Decrease.

1. What is Compound Subtraction? and what is Compound Division?

2. What is the rule for Compound Subtraction, in dollars and cents? also the proof?

3. What is the rule for Compound Division, in dollars and cents? also the proof?

4. What is the rule for Compound Subtraction, in money, weight, &c.? also the proof?

5. What is the general rule for Compound Division, in money, weight, &c.? also the proof?

6. What is the rule when the divisor is not over 12?

7. What is the rule when the divisor is a composite number?

8. To which of the rules respectively do the terms apply, and what is their import, viz.—Dividend, Minuend, Divisor, Subtrahend, Remainder, Difference, and Quotient? and where are their several positions?

SUPPLEMENT TO MULTIPLICATION.

To multiply by a mixed number; that is, a whole number, and a fraction joined to it, which fraction is expressive of a part of one whole number, as 3½, 4¼, 5¼, &c.

DUODECIMALS, OR TWELVES,

USUALLY CALLED

CROSS MULTIPLICATION.

This is a rule used by artificers, in computing the contents of their work. It is founded on the principle of twelves, in the fractional parts of a foot, viz. for inches, seconds, thirds, &c. as in each of these denominations one is carried for every twelve.

Inches are often called primes; parts of inches, seconds; and parts of parts, thirds, &c.

Sexigessimals, or sixties, might be conducted on the same principles, except carrying by 12, carry by 60, for the fractional parts of an hour; viz. for minutes, seconds, thirds, &c.

Rule for Duodecimals.

1. Write the multiplier under the corresponding denominations in the multiplicand.

2. Begin with the highest denomination in the multiplier, and multiply it into each term of the multiplicand, commencing with the lowest term; and write the result of each multiplication under its respective term; observing to carry one for every 12, from each lower denomination to the product of the next superior.

3. In like manner multiply the inches, or primes, in the multiplier, into all the denominations of the multiplicand, and place the result of each term one remove to the right-hand of those in the multiplicand, carrying by 12 as before.

4. Proceed with the seconds in the multiplier in the same

[illegible]

[illegible]

[illegible]

How many square feet in a board, 14 *ft.* 8 *in.* long, and 2 *ft.* 2 *in.* wide?

Ft.	in.		
14	8		
2	2		
29	4		
2	5	4	
31	9	4	*Ans.*

How many square feet in 10 boards, each 15 *ft.* 3 *in.* long, and 1 *ft.* 10 *in.* wide?

Ft.	in.		
15	3		
1	10		
15	3		
12	8	6	
27	11	6	
		10	
279	7	0	*Ans.*

How many solid feet of bark in a load 7 *ft.* 3 *in.* long, 3 *ft.* 4 *in.* wide, and 6 *ft.* 4 *in.* high?

ft.	in.	
7	3	
3	4	
21	9	
2	5	0
24	2	0
6	4	
145	0	0
8	0	8
153	0	8

Ans. 1*cd.* 25 *ft.* 0 *in.* 8″

[illegible] and

[illegible] be
[illegible] tained
[illegible]

[illegible] pli-
[illegible] ?
[illegible]
[illegible] ches,
[illegible] ble ?
[illegible] pro-
[illegible] Mea-

REDUCTION.

Reduction is the bringing or changing of numbers from one denomination to another, without altering their value: as a pound reduced to farthings; or a hundred weight to ounces; yet they retain their respective values equally as before, although their respective *denominations* have undergone a *change*. Because 960 farthings occupy more places of *figures* than unity or one, under the denomination of pounds, yet their intrinsic value is the same. So also of 1792 ounces and 1 hundred weight.

There are two kinds of Reduction, viz.—Descending and Ascending. Reduction Descending is, when larger denominations are brought, or descend into smaller; as shillings into pence; and feet into inches, &c. This is accomplished by Multiplication.

Reduction Ascending is, when smaller denominations are brought, or ascend to larger; as shillings to pounds, ounces to pounds, quarters, &c. This is accomplished by Division.

Rule.

Multiply the higher denomination given, by so many of the next lower, as make one of the greater; and to that product add the figures of the same denomination, if any, in the given sum; and thus proceed until it is brought as low as the question requires.

Proof.—Change the order of the question, and divide the last product by the last multiplier. The remainders severally, if any, in each division are of the same denomination as the respective dividends.

When smaller denominations are to be brought into larger, as pence into shillings, gallons into hogsheads, &c.

Rule.

Divide the given sum by as many of its own denominations as will make one of the next greater; and so on from one denomination to another, until it is brought or ascends to the denomination required.

Should there be a remainder in dividing, it will be of the same name as the dividend whence it arises.

Note. A given sum of different denominations, to be reduced, is of the nature of a fraction, and therefore is a broken number. The different denominations, subordinate to the highest, clearly show they are but parts of one unit of the highest. If £2. 17*s.* 6*d.* 2 *qrs.* were the given sum, the shillings, pence, and farthings of it are only the parts of one pound, and not a whole pound, or unit, to be added to the two pounds. Pounds *increase* in a tenfold ratio; but they *decrease* by 20, 12, and 4, which constitute their minor denominations. From minor denominations they *increase* by 4, 12, and 20, as in the table of money; and, in every other table, according to the respective quantities of its given denominations. Hence the minor denominations of every table of money, weights, measures, &c. are only broken numbers, or parts of *one*, of their highest respective denominations. From these considerations, it would not have been improper to have treated at large of *Fractions*, previous to Reduction. But as this would be a deviation from the long established usages of authors and compilers of Arithmetic, I have pursued the old method in this particular. Yet the suggestions already given are essential to the learner, to aid him in the acquisition of these rules. It is indispensably necessary to the student, carefully to observe the given sum; in what denomination it stands; to what denomination it is to be reduced; whether by descending, or ascending; and what are the several denominations by which to multiply or divide the given sum, as the case may require, in order to obtain the desired answer. This renders it important to understand thoroughly the different tables of weights, measures, &c., and in pursuing these directions the work is easily and speedily accomplished.

Note.—Pounds × 20, give shillings; shillings × 12, give pence; pence × 4, give farthings. So 1 Cwt. × 4, gives quarters; quarters × 28, give pounds; pounds × 16, give ounces; and ounces × 16, give drams. By inverting these orders, by division, the numbers ascend to their primitive state. The same would be true of every other table of measure, &c.

REDUCTION DESCENDING.

Table of Money.

Example.—In £28 12*s.* 4*d.* 2*qr.* how many farthings?

	£ *s.* *d.* *qr.*	
	28 12 4 2	
	20	added in the 12 shillings.
	572	shillings.
	12	pence in 1*s.* and add the 4*d.*
	6868	pence.
	4	farthings, and add the 2 far
Ans.	27474	farthings sought in the quest.

2. In £36 11*s.* 9*d.* 1 *qr.*, how many farthings? *Ans.* 35125.
3. In £44 13*s.* 7*d.* 2 *qrs.* how many farthings? *Ans.* 42894.
4. In £71 5*s.* 11*d.*, how many pence? *Ans.* 17111.
5. In £94 15*s.*, how many shillings? *Ans.* 1895.
6. In £210, how many shillings? *Ans.* 4200.
7. In 16*s.* 7*d.*, how many pence and farthings? *Ans.* 199*d.* 796*qrs.*
8. In $582, at 6*s.* each, how many farthings?—*Ans.* 167616.
9. In $582, at 8*s.* each, how many farthings?—*Ans.* 223488.
10. In 36 guineas, at 28*s.* each, how many pence? *Ans.* 12096.
11. In 42 pistoles, at 22*s.* each, how many shillings and pence? *Ans.* 924*s.* 11088*d.*
12. In 26 half Johannes, at 48*s.* each, how many shillings, sixpences, and threepences?—*Ans.* 1248*s.* 2496*six.* 4992*threep.*

13. In 125 French crowns, at 6*s.* 8*d.* each, how many pence and farthings? *Ans.* 10000*d.* 40000*qrs.*

Rule II.

REDUCTION ASCENDING.

In 35125*qrs.*, how many pence, shillings, and pounds?

Farthings in a penny	4)35125
Pence in a shilling	12) 8781 1*qr.*
Shillings in a pound	20) 731 9*d.*
	£ 36 11*s.*

Ans. £36 11*s.* 9*d.* 1*qr.*

2. In 42894*qrs.*, how many pence, shillings, and pounds? *Ans.* £44 13*s.* 7*d.* 2*qr.*
3. In 17111*d.*, how many shillings and pounds? *Ans.* £71 5*s.* 11*d.*
4. In 1895*s.*, how many shillings and pounds?—*Ans.* £94 15*s.*
5. In 4200*s.*, how many pounds? *Ans.* £210.
6. In 199*d.*, how many pence and shillings? *Ans.* 16*s.* 7*d.*
7. In 167616*qrs.*, how many dollars at 6*s.* each?—*Ans.* $582.
8. In 223488*qrs.*, how many dollars at 8*s.* each?—*Ans.* $582.
9. In 12096*d.*, how many guineas, at 28*s.* each?—*Ans.* 36*g.*
10. In 11088*d.*, how many pistoles, at 22*s.* each?—*Ans.* 42*p.*
11. In 2496 sixpences, how many half Johannes, at 48*s.* each? *Ans.* 26 half Johannes.
12. In 4992 threepences, how many half Johannes, at 48*s.* each? *Ans.* 26 half Johannes.
13. In 10000 pence, how many French crowns, at 6*s.* 8*d.* each? *Ans.* 125*crs.*
14. In 40000*grs.* how many French crowns, at 6*s.* 8*d.* each? *Ans.* 125 *Crowns.*
15. In 18144*d.*, how many moidores, at 36*s.* each?—*Ans.* 42.
16. In 72576*qrs.*, how many moidores, at 36*s.* each? *Ans.* 42 *Moidores.*

REDUCTION OF FEDERAL MONEY.

THIS is readily accomplished without using at length, either Multiplication or Division. As decimals are framed on the principles of tens, as whole numbers are, it is necessary only to *bear in mind*, that dollars are made cents by annexing two ciphers, which is equivalent to multiplying the dollars by 100; and dollars become mills, by annexing three ciphers; but in either case, no separating point is used. So, if a given sum already stand in dollars and cents, it is only to take away the separatrix, and they all become cents: or if dollars, cents, and mills; remove all the separating points, and they all become mills. If the sum is given in cents, annex one cipher, and they all become mills. To reverse the order, mills are made cents, by separating one figure from the right-hand for mills, which is equivalent to dividing by 10; and cents are made dollars, by separating two figures from the right-hand for cents, which is the same as dividing by 100. Thus dollars will stand at the left-hand of the separatrix, and cents and mills at the right. In order to distinguish dollars from cents, when they are connected in a given sum, the *separatrix* is indispensably requisite. It is left optional with the learner whether to separate cents and mills, or not.

Examples.

Reduce $56,67 into cents.	*Ans.* 5667 *Cents.*
Reduce $42,15 into mills.	*Ans.* 42150 *Mills.*
Reduce $34,00 into cents.	*Ans.* 3400 *Cents.*
Reduce $5,09 into mills.	*Ans.* 5090 *Mills.*
Reduce $110 into mills.	*Ans.* 110000 *Mills.*
Reduce 51676 mills into dollars.	*Ans.* $51,67,6.
Reduce 31702 mills into dollars.	*Ans.* $31,70,2.
Reduce 25000 cents into dollars.	*Ans.* $250,00.
Reduce 5375 mills into dollars.	*Ans.* $5,37,5.
Reduce 18005 mills into dollars.	*Ans.* $18,00,5.

In ten dollars and six cents, how many mills?—*Ans.* 10060 *M.*

In twenty dollars and two mills, how many mills?
Ans. 20002 *Mills.*

In five hundred and twenty mills, how many dollars?

Ans. \$0,52,0.

In seven hundred and three cents, how many dollars?

Ans. \$7,03.

Troy Weight.

In 20 ingots of gold, each weighing 2*lb.* 11*oz.* 16*pwt.* 16*grs.* how many grains?

```
 lb. oz. pwt. grs.
   2  11  16  16
  12 ounces in a lb.
  --
  35 ounces.
  20 pennyweights in an ounce.
 ---
 716 pennyweights.
  24 grains in one pennyweight.
 ---
 2880
1432
-----
17200 grains.
-----
```

```
          Proof. pwts.grs.                 pwt.
Div. by 24)17200(716 16 left.     20)716(35 16 left.
           168                        60
           ---                        ---
            40                        116
            24                        100
            --                        ---
           160                         16
           144                         --
           ---
            16                          oz.
            --                    12)35(2 11 left.
                                     24
                                     --
                                     11
                                     --
```

Ans. 2*lb.* 11*oz.* 16*pwt.* 16*grs.*

How many grains are there in a silver tankard, that weighs 2*lb.* 10*oz.* 18*pwt.* 12*grs.*? *Ans.* 16764 *grs.*

In 3*lb.* 9*oz.* 19*pwt.* 15*grs.* how many grains? *Ans.* 22071 *grs.*

In 16764 *grains*, how many pounds, &c.? *Ans.* 2*lb.* 10*oz.* 18*pwt.* 12*grs.*

In 22071 *grains*, how many pounds? *Ans.* ℔3 9 19 15.

Avoirdupois Weight.

In 76*cwt.* 2*qrs.* 21*lb.* 14*oz.* how many ounces?

```
Cwt. qrs. lb. oz.          Proof. 16)137438(8589—14oz.
 76   2   21  14                     128             [left.
  4                                  ---
 ---                                  94
 306 qrs.                             80
  28                                  ---
 ---                                  143
 2469                                 128
 612                                  ---
 ---                                   158
 8589 lbs.                             144
   16                                  ---
 -----                                  14
 51548                                  --
 8589
 ------
 137438 oz.
 ------

28)8589(306—21lb. left.      4)306(76—2qrs. left.
   84                           28
   ---                          --
    189                         26
    168                         24
    ---                         --
     21                          2
     --                          -
```

In 2*ts.* 10*cwt.* 1*qr.* 17*lb.* 12*oz.*, how many ounces? *Ans.* 90332*oz*

In 137438*oz.*, how many hundredweight, &c.? *Ans.* 76*cwt.* 2*qr.* 21*lb.* 14*oz.*

In 90332*drs.*, how many hundredweight?

Ans. 2*t.* 10*cwt.* 1*qt.* 17*lb.* 12*oz.* 14*drs.*

Apothecaries' Weight.

In 7lb. 9℥. 7ʒ. 2℈. 75*grs.*, how many pounds, ounces, &c.?

```
lb.  ℥.  ʒ.  ℈. grs.
 7   9   7   2  15
12
--
93
 8
---
751
  3
----
2255
  20
-----
45115
-----
```

```
Proof. 20)45115(2255—15 left.        3)2255
          40                          ------
          --                          8) 751  2 left.
           51                         ------
           40                        12)  93  7 left.
           --                         ------
           111                             7  9 left.
           100                        ------
           ---
            115
            100
            ---
             15
             --
```

Ans. 7*lb.* 9*oz.* 7*drs.* 2*scr.* 15*grs.*

In 20555*grs.*, how many pounds Apothecary Weight?

Ans. 3*lb.* 6*oz.* 6*drs.* 1*scr.* 15*grs.*

In 45115*grs.*, how many pounds Apothecary Weight?

7*lb.* 9*oz.* 7*drs.* 2*scr.* 15*grs*

In 3*lb.* 6*oz.* 6*drs.* 1*scr.* 15*grs.*, how many grains?

Ans. 20555*grs*

Dry Measure.

In 36 *bushels*, how many pecks, quarts, and pints?
Ans. 144*pks.* 1152*qts.* 2304*pts.*

In 67*bus.* 3*pk.* 5*qt.* 1*pt.*, how many pints? *Ans.* 4347*pts.*

A man would ship 1925 *bushels* of wheat in tierces, containing 6 *bushels* and 1 *peck* each; how many tierces would he require?
Ans. 308 *tierces.*

In 2304 *pints*, how many bushels? *Ans.* 36 *bushels.*

In 4347 *pints*, how many bushels, &c.?
Ans. 67*bus.* 3*pcs.* 5*qts.* 1*pt.*

In 308 *tierces*, each containing 6 *bushels*, 1 *peck*, how many bushels would they contain? *Ans.* 1925 *bushels.*

Wine Measure.

In 8 *tuns* of wine, how many hogsheads, gallons, and quarts?
Ans. 32 *hogsheads*, 2016 *gallons*, 8064 *quarts*

In 14300 *pints* of wine, how many hogsheads?
Ans. 28 *hogsheads*, 23 *gallons*, 2 *quarts.*

What number of bottles, containing 6 *gills* each, can be filled with a barrel of cider? *Ans.* 168.

In 8064 *quarts* of wine, how many tuns? *Ans.* 8.

In 2016 *gallons* of wine, how many tuns? *Ans.* 8.

In 28 *hogsheads*, 23 *gallons*, 2 *quarts*, how many pints of wine? *Ans.* 14300.

How large a cask would 168 *bottles*, each containing 6 *gills*, fill? *Ans.* 1 *barrel.*

How many pints, quarts, and two quarts, of each an equal number, may be filled from a tun of wine? *Ans.* 288.

Long Measure.

In 62 *miles*, how many rods? *Ans.* 19840

In 76 *yards*, how many barley corns? *Ans.* 8208

How many barley corns will it take to reach from Philadelphia to Boston, it being 343 *miles* *Ans.* 65197440

How many inches round the globe, it being 360 *degrees*?
Ans. 1585267200.

How many times will a wheel, 15 *feet* 6 *inches* in circumference, turn round in going from New-York to New-Haven, it being 90 *miles*? *Ans.* $30658\frac{12}{186}$.

In 15840 *yards*, how many miles and leagues?—*Ans.* 9*m.*=3*l.*

Land or Square Measure.

In 317 *acres*, 3 *roods*, 32 *rods*, how many square rods, perches, or poles? *Ans.* 50872.

In 26974 square *rods*, how many acres?—*Ans. A.*168 2 14.

If a piece of land contain 74 *acres* and 3 *roods*; and an inclosure of 25 *acres* and 2 *roods* be taken out of it, how many rods are there left? *Ans.* 7880.

Cloth Measure.

In 72 *yards*, how many nails? *Ans.* 1152.

In 234 *yards*, 3 *quarters*, 2 *nails*, how many nails?—*Ans.* 3758.

In 61 *ells English*, how many quarters? *Ans.* 305.

In 72 *ells Flemish*, how many quarters? *Ans.* 216.

In 56 *ells French*, how many quarters? *Ans.* 336.

In 123 *ells English*, how many ells Flemish? *Ans.* 205.

In 1920 *nails*, how many yards, ells Flemish, and ells English?
Ans. 120 *yards*, 160 *ells Flemish*, 96 *ells English.*

In 3758 *nails*, how many yards?—*Ans.* 234*yds.* 3*qrs.* 2*n.*

Solid Measure.

In 12 *tons* of hewn timber, how many solid inches?
Ans. 1036800.

In 20 *tons* of round timber, how many inches?—*Ans.* 1382400.

In 27 *cords* of wood, how many solid feet? *Ans.* 3456.

In 4864 *feet* of wood, how many cords? *Ans.* 38.

Grindstones are sold by the cubic foot, commonly called a stone, the contents of which are found thus:

Rule.

Add half of the diameter to the whole diameter, then multiply this sum by half of the diameter, and the product by the thickness: divide the last product by 1728, the cubic inches in a foot, and the quotient will be the contents in feet, and the remainder cubic inches.

Note.—The diameter and thickness will be taken in inches.

Examples.

1. How many cubic feet in a grindstone, 36 *inches* in diameter, and 4 *inches* thick?

$36+18\times18\times4=3888$. Then,

```
1728)3888(2 feet.
     3456
     ----
      432
        4 quarters.
     ----
1728)1728(1 quarter.
     1728
     ----
```

Ans. 2*ft.* 1*qr.*

2. What are the contents of a grindstone, 42 *inches* in diameter, and 5 *inches* thick?

$$42+21\times21\times5\div1728=3\tfrac{1431}{1728}\ \textit{feet}.\ \textit{Ans.}$$

3. How many cubic feet in a grindstone, 28 *inches* in diameter, and $3\frac{1}{2}$ *inches* thick?

$$28+14\times14\times3\tfrac{1}{2}\div1728=1\tfrac{330}{1728},\ \textit{cubic feet}.\ \textit{Ans.}$$

Time.

In 32 *weeks*, how many days, hours, minutes and seconds?
Ans. 224*d.* 5376*h.* 322560′ 19353600″.

In 245 *days*, 18 *hours*, 36 *minutes*, how many minutes?
Ans. 353916.

In 353916 *minutes*, how many weeks?—*Ans.* 35*w.* 0*d.* 18*h.* 36′.

How many days from the birth of Christ to Christmas 1827, allowing the year to contain 365 days and 6 hours?

Ans. 667311 *days*, 18 *hours*.

If a person's age be 50 *years*, how many minutes old is he, calling the year 365 days? *Ans.* 26280000.

Circular Motion.

In 8*s.* 18°. 45′. 25″., how many seconds? *Ans.* 931525.

In 5*s.* 12°. 36′., how many minutes? *Ans.* 9756.

In 931525 *seconds*, how many signs, &c.?—*Ans.* 8*s.* 18°. 45′. 25″.

In 9756 *minutes*, how many signs, &c.? *Ans.* 5*s.* 12°. 36′.

In 265376 *seconds*, how many signs, &c.

Ans. 2*s.* 13°. 42′. 56″

Practical Questions.

In £375 15*s.* 8*d.*, how many pence, twopences, threepences, fourpences, sixpences, and shillings, are contained; and of each an equal number?

1+2+3+4+6+12=28. *Ans.* 3221 of each.

A person borrowed 32*gui.* at 28*s.* each; 36 *E. c.*, at 6*s.* 8*d.*; 20*pis.*, at 22*s.*; and $140 at 6*s.* each; how many moidores, at 36*s.* each will replace the loan? *Ans.* 71.

A merchant purchased three pieces of cloth; one contained 28*yds.*; one 28*e. E.*; one 28*e. Fl.*; how many ells French were there in the whole? *Ans.* 56 *Ells French*.

Two ships, one 98*ft.* 4*in.* in length, the other 90*ft.* long, sail for Liverpool, a distance of 3000 miles. How many more times will the shorter ship sail her length, than the longer one, in reaching the destined port? *Ans.* 14916.

If the national debt of the United States amount to sixty-three millions of dollars; how long would it take one to count over the sum, counting fifty dollars a minute, and employing himself 8*hrs.* daily, until it was completed?—*Ans.* 7*yrs.* 70*dys.*

Suppose a ship homeward bound from Madeira, laden with 310*pps.* of wine, 350*hhds.* and 126 *half hhds.*; how many gallons in all; and allowing every pint to be a pound, what was the burden of the ship?

Ans. 65079*gal.* and the burden was 232*t.* 8*cwt.* 2*qrs.*

If a cannon ball were to fly one mile in 7½ *seconds*; how long would it take to reach to the sun, which is distant 96,000,000 of miles from the earth? *Ans.* 22*yrs.* 303*d.* 8*hrs.*

If the sun travels through the twelve signs of the Zodiac in a year; how many degrees, minutes, and seconds would there be?

Ans. 360°. 21600′. 1296000″

Questions relative to Reduction.

1. What is Reduction?
2. Is the intrinsic value of an object changed by Reduction?
3. How many kinds of Reduction, and what are they called?
4. What is meant by Descending and Ascending?
5. What is the rule for Reduction Descending, and why the effect produced?
6. What is the rule for Reduction Ascending, and why its effect?
7. By what gradation is Reduction Ascending, or Descending accomplished, of any table of Money, Weight, or Measure?
8. How are Doubloons, Moidores, Guineas, Pistoles, and Crowns reduced to Shillings, Pence, and Farthings?
9. How are Dollars reduced to Cents, and Mills? and Mills and Cents reduced to Dollars?
10. What is the rule for finding the cubic feet in a Grindstone?
11. If it were required to find how many Shillings, Ninepences, Sixpences, Threepences, Twopences, and Pence there are in £1000, and of each an equal quantity, how could it be ascertained?
12. Which contains the most ounces 1800*lb.* Troy, or 1350*lb.* Avoirdupois?

FRACTIONS.

Note.—Before entering upon a treatise of Fractions, it may be useful to premise a few considerations, which will tend to assist the learner in a more comprehensive view of their nature and design. Were the derivation of words, and their radical significations well understood, it would be seen that the *technical* terms, used in the arts and sciences generally, are not merely *arbitrary* words, adopted without any meaning of their own; but that each is clearly explanatory of its own use and import. If this were realized, much labour might be saved, and far more clear and distinct ideas obtained from this source. The very *terms* used in Fractions, are fully explanatory of their significations and uses. The word *Fraction*, I have before observed, is a *broken* number, and not unity, or one whole number. It is derived from "fractus,"* *broken*. We have already noticed in Division, that where the divisor did not exactly measure the dividend, and of course a remainder was left, we had a specimen of a broken number; viz. a remainder, which is always less than the divisor; for if it were greater, then the divisor could have been obtained *one* or *more* times over the number of the *quotient* already produced. Here is the origin of fractions; they being only a broken number or parts of one whole, they are mere fragments *broken*, as it were, from it. But although they are broken parts, they are nevertheless valuable; and it is important that their value should be ascertained and saved. With equal propriety might a watch be thrown away as of no value, because its chain, or one of its wheels was broken, and it was therefore severed into parts. By putting these parts together, in a skilful manner, it might be

restored to a *whole* again, and its value remain undiminished. But without *skill* in combining the parts of broken numbers, no benefit could arise, or object be effected. This is therefore to be accomplished solely by a thorough knowledge of the *nature* and *operations* of broken numbers, and has rendered a treatise on this subject indispensable. It may be proper here to remark also, that the terms Numerator and Denominator, which are abundantly used in Fractions, are clearly expressive of their significations.

Numerator, is derived from the participle "*numeratus*,"* which signifies *numbered*. The Numerator, which answers to the remainder after division, shows how many parts are to be taken from the divisor; or the *numbered* parts in relation to the parts of the divisor. The term Denominator, is derived from "*denominatus*," signifying, denominated, named, or distinguished. If the divisor includes a given number, whether this number is composed of a greater or less quantity of figures, still it is but one divisor given, to divide a given dividend; and as these parts are taken collectively in a divisor, or one given sum, in dividing, when there is a remainder after division, this remainder less than the dividend, is placed over it; but the divisor is *distinguished* into parts, which are expressed by the number of figures of which it is composed, and it imports *denominated, named*, or distinguished into the parts of its whole, and thus is called Denominator. Hence the Numerator denotes the *numbered* parts of the remainder; and the Denominator denotes the denominated, or distinguished parts of the Divisor. If 7 parts are *numbered* as remainder, and the divisor is composed of the number 9, and the 9 constitutes but one sum; yet, when placed under the Numerator, fraction-wise, it is *denominated*, or *distinguished* into 9 parts; as $\frac{7}{9}$; meaning, that 7 parts are to be taken from 9 parts, which consequently cannot be equal to a whole, or $\frac{9}{9}$.

Fractions, or *broken* numbers, are expressions for any assignable, or *numbered* part of a unit, or of a whole number. They are usually divided into two kinds; viz. *Vulgar* and *Decimal*. A

* Fractus, numeratus, and denominatus, are respectively Latin perfect participles.

Vulgar Fraction is expressed by two numbers placed one above the other, with a line drawn between them, thus; $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, &c. and signify two-thirds, three-fourths, and four-fifths.

The figures above the line, are called Numerators; those below it are called Denominators.

The Denominator (which is the divisor in division,) shows how many parts the integer, or whole quantity, is divided into: the Numerator (which is the remainder after division,) shows how many of those parts are meant by the fraction.

It is evident from the manner of representing fractions, that when the Numerator only of a given fraction is increased, the value of the fraction becomes greater; but when the Denominator only is increased, the value becomes less. It is hence clear, that if the Numerator and Denominator are both equally increased, or both equally diminished, the value, in either case, is not altered: therefore, if both are multiplied by any number whatever, or divided by any number which will exactly measure both, other fractions of equal value will be obtained. It is hence apparent, that every fraction can be expressed in a variety of forms, all which have the same signification.

Vulgar Fractions are distinguished by various names, viz.—single, or simple, proper, improper, compound, and mixed.

A *single*, or *simple* fraction, is an individual fraction, expressed fractionwise, unconnected with any other fractions. It does not necessarily imply, that the Numerator is greater or less than the Denominator; thus $\frac{6}{7}$, or $\frac{7}{6}$, would each be either single, or simple fractions.

A *proper* fraction is, when the Numerator is less than the Denominator; as $\frac{5}{6}$, $\frac{11}{12}$, &c.

An *improper* fraction is, when the Numerator is greater than the Denominator; as $\frac{9}{8}$, $\frac{24}{3}$, &c.

A *compound* Fraction, is the fraction of a fraction, connected by the word *of*; as $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$, or $\frac{1}{2}$ of $\frac{5}{7}$ of $\frac{6}{8}$, &c.

A *mixed* number, is composed of a whole number and a fraction; as $5\frac{1}{3}$, $6\frac{5}{7}$, $8\frac{7}{11}$, &c.

Any whole number may be expressed like a fraction, by drawing a line under it, and putting one for the Denominator; as $8=\frac{8}{1}$, $5=\frac{5}{1}$, $12=\frac{12}{1}$, &c.

[illegible]

[illegible]

[illegible] can
[illegible] the
[illegible]

2. What is the least common multiple of 4, 5, 6, 10?

5)4, 5, 6, 10
2)4, 1, 6, 2
2, 1, 3, 1

$5\times2\times2\times3=60$ *Ans.*

3 What is the least number that 3, 4, 5, 8, 12, will measure?
Ans. 120.

3. What is the least number that can be divided by the 9 digits, without a remainder? *Ans.* 2520.

Reduction of Vulgar Fractions,

Is bringing them out of one form into another, to prepare them for the operations of Addition, Subtraction, &c.

Case I.

To abbreviate, or reduce Fractions to the lowest terms.

Rule.

1. Find a common measure, by dividing the greater term by the less, and the last divisor by the last remainder, and continuing the division down, until nothing remains. The last divisor, which left no remainder, is the common measure. If there are more than two numbers given, of which to find the common measure; proceed as with the two numbers above; and with the common measure of these, divide another number as before, and should there be a fourth number, divide it by the common measure last found; and this last common measure will divide all the given sums, without a remainder. Should it happen, that no number greater than one would divide the given sums; then these sums are said to be primes to each other, and cannot be reduced any lower. Or, if one can locate upon a number, that will divide both parts of the fraction, without any remainders, it will shorten the work.

2. When the common measure is found, reduce both terms of the fraction by it, and the quotients will form the fractions

required. Ciphers on the right-hand of both terms may be rejected.

1. Reduce $\frac{66}{72}$ to its lowest terms.

```
66)72(1
   66      last divisor.
   --
    6)66(11
      66
      --
       0 Remainder.
      --
```

Common measure $6\left|\begin{array}{c|c}66 & 11 \\ \hline 72 & 12\end{array}\right.$ *Ans.* $\frac{11}{12}$.

2 Reduce $\frac{175}{225}$ to its lowest terms.

```
175)225(1
    175
    ---
     50)175(3
        150
        ---
         25)50(2
            50
            --
             0 Remainder.
            --
```

$25\left|\begin{array}{c|c}175 & 7 \\ \hline 225 & 9\end{array}\right.$ *Ans.*

3. Reduce $\frac{204}{238}$ to its lowest terms. *Ans.* $\frac{6}{7}$.

4. Reduce $\frac{1144}{1404}$ to its lowest terms. *Ans.* $\frac{22}{27}$.

Note.—In all answers, the fractions should be reduced as low as possible.

Case II.

To reduce a mixed number to its equivalent improper fraction.

Rule.

Multiply the whole number by the denominator of the frac-

tion, and to that product add the numerator, and this sum written over the denominator, will give the fraction required.

Note.—Every mixed number must necessarily form an improper fraction.

1. Reduce $12\frac{3}{5}$ to its equivalent improper fraction.

$$\begin{array}{c} 12\frac{3}{5} \\ 5 \\ \hline 63 \\ \hline 5 \end{array} \qquad 12\times5+3=\frac{63}{5}$$

2. Reduce $25\frac{7}{8}$ to its equivalent improper fraction. *Ans.* $\frac{207}{8}$.

3. Reduce $47\frac{11}{12}$ to its equivalent improper fraction. *Ans.* $\frac{575}{12}$.

4. Reduce $51\frac{3}{8}$ to its equivalent improper fraction. *Ans.* $\frac{411}{8}$.

Case III.

To reduce an improper fraction to its equivalent whole, or mixed number.

Rule.

Divide the numerator by the denominator, and the quotient will be the whole number, and the remainder will be the numerator to the denominator.

1. Reduce $\frac{58}{8}$ to a mixed number.

$$\begin{array}{l} 8)58(7\frac{2}{8}\ \textit{Ans.} \\ \quad 56 \\ \quad \overline{\ \ 2\ \ } \end{array}$$

2. Reduce $\frac{99}{12}$ to a mixed number. *Ans.* $8\frac{3}{12}$ or $8\frac{1}{4}$.

3. Reduce $\frac{411}{8}$ to a mixed number *Ans.* $51\frac{3}{8}$.

4. Reduce $\frac{7654}{9}$ to a mixed number. *Ans.* $850\frac{4}{9}$.

5. Reduce $\frac{81}{9}$ to a whole number. *Ans.* 9.

CASE IV.

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.

Multiply the whole number by the given denominator, and place the product over the said denominator, and it will form the fraction required.

1. Reduce 8 to a fraction, whose denominator is 4.

$$8\times4=\frac{32}{4}. \textit{ Ans.}$$

2. Reduce 15 to a fraction, whose denominator is 9. *Ans.* $\frac{135}{9}$.

3. Reduce 25 to a fraction, whose denominator is 4. *Ans.* $\frac{100}{4}$ or $\frac{25}{1}$

4. Reduce 50 to a fraction, whose denominator is 6. *Ans.* $\frac{300}{6}$ or $\frac{50}{1}$.

CASE V.

To reduce a compound fraction to an equivalent simple one.

1. Reduce whole and mixed numbers to improper fractions.

2. Multiply all the numerators together for a new numerator, and all the denominators for a new denominator, and they will form the fraction required.

1. Reduce $\frac{2}{8}$ of $\frac{3}{4}$ of $\frac{7}{8}$ to a simple fraction.

$$\frac{2\times3\times7=42}{8\times4\times8=256}=\frac{21}{128} \textit{ Ans.}$$

2. Reduce $6\frac{2}{3}$ of $\frac{4}{5}$ of [illegible] $\frac{2}{9}$, to a simple fraction.

$$\begin{array}{c} 6\frac{2}{3} \\ 3 \\ \hline 20 \\ \hline 3 \end{array}$$

$\frac{20}{3}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of $\frac{2}{9}$ $\qquad \dfrac{20\times4\times7\times2=1120}{3\times5\times8\times9=1080}=\dfrac{28}{27}$ *Ans*

3. Reduce $\frac{2}{5}$ of $\frac{4}{7}$ of $\frac{5}{9}$ of $\frac{7}{13}$ to a simple fraction.

$\frac{280}{4095}=\frac{8}{117}$ *Ans*

4. Reduce $\frac{15}{1}$ of $\frac{4}{7}$ of $\frac{9}{11}$ to a simple fraction. $\frac{540}{77}$ *Ans*

5. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of 12 to a simple fraction.

$$\begin{array}{r|l} 2\times3\times12=72 & 6 \\ 3\times4\times\ 1=12 & \overline{1} \end{array} \textit{ Ans.}$$

Note. If the denominator of a compound fraction be equal to the numerator of another, they may both be expunged; or wherever the numerator of one balances the denominator of another, expunge them; and multiply the remaining numbers continually together as before; and the answer will be obtained; but in lower terms, yet of equal value.

6. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{7}$ of $\frac{8}{9}$ to a simple fraction.

$$\begin{array}{r} 3\times8=24 \\ 7\times9=\overline{63} \end{array} \textit{ Ans.}$$

7. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

Ans. $\frac{1}{5}$; or, if all the numerators and denominators were multiplied, and then reduced to their lowest terms, the answer would be the same.

Thus, $\dfrac{1\times2\times3\times4=24}{2\times3\times4\times5=120}\div$ by $24 \left| \dfrac{24=1}{120=5} \right.$ *Ans.*

Note.—Numerators and denominators may frequently be contracted by taking their aliquot parts.

8. Reduce $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{3}{8}$ of [illegible] to a simple fraction.

Cancelling they stand $\div 2 | \frac{2}{1}$ of $\frac{1}{5}$ of $\frac{8}{8} | \frac{1}{1}$ of $\frac{1}{3}$ of $\frac{3}{8} = \frac{8}{27}$ *Ans.*

Case VI.

To reduce Fractions of different Denominations to equivalent Fractions, having a Common Denominator.

Rule.

1. Reduce whole and mixed numbers to improper fractions
2. Multiply each numerator into all the denominators except its own, for a new numerator; and all the denominators into each other continually, for a common denominator; this written under the several new numerators, will give the fractions required.

Note.—The reason of this rule is obvious. The numerators and denominators of each fraction are, in the course of the operation, multiplied by the same numbers, and therefore their value is not altered. In this manner, the different given denominators are made to be of the same quantity, although increased; and the given numerators are thus enhanced, so as to bear the same proportions to their respective new denominators, as before.

1. Reduce $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$ to equivalent fractions, having a common denominator.

The first numerator	$2\times5\times6\times8=480$
2d numerator	$4\times3\times6\times8=576$
3d numerator	$5\times3\times5\times8=600$
4th numerator	$7\times3\times5\times6=630$
Common denominator	$3\times5\times6\times8=720$

New Nu.	2d.	3d.	4th.	
$\frac{480}{720}$	$\frac{576}{720}$	$\frac{600}{720}$	$\frac{630}{720}$	New Denominator.

$24 \left| \frac{48}{72} \right| \frac{0}{0} \left| \frac{2}{3} \right|$ $\quad 144 \left| \frac{576}{720} \right| \frac{4}{5}$ $\quad 120 \left| \frac{600}{720} \right| \frac{5}{6}$ $\quad 90 \left| \frac{630}{720} \right| \frac{7}{8}$

2. Reduce $\frac{3}{7}$, $\frac{5}{9}$, $\frac{7}{11}$.

$$\left.\begin{array}{l} 3\times9\times11=297 \\ 5\times7\times11=385 \\ 7\times7\times\ 9=441 \end{array}\right\}\text{New Numerator.}$$
$$7\times9\times11=693 \quad \text{New Denominator.}$$

$$\frac{\text{New Numerator}}{\text{New Denominator}} \quad \frac{297}{693} \quad \frac{385}{693} \quad \frac{441}{693} \text{ Ans.}$$

3. Reduce $4\frac{1}{2}$, $\frac{3}{5}$ and $\frac{2}{3}$ of $\frac{1}{3}$.

$$\begin{array}{l} 4\frac{1}{2} \\ 2 \\ \hline \end{array}$$

$$\frac{9}{2} \quad \frac{3}{5} \quad \frac{2}{9} \qquad \frac{405}{90} \quad \frac{54}{90} \quad \frac{20}{90} \text{ Ans.}$$

4. Reduce $\frac{5}{6}$, $\frac{2}{5}$, $\frac{4}{7}$, $\frac{5}{6}$

$$\frac{1050}{1260} \quad \frac{504}{1260} \quad \frac{720}{1260} \quad \frac{1050}{1260} \text{ Ans.}$$

The foregoing is a general rule for reducing fractions to a common denominator; but as much labour is saved by keeping the fractions in the lowest terms possible, the following rule may be considered preferable.

Rule II.

For reducing fractions to the least common denominator. Find the least common multiple (by first rule in fractions) of all the denominators of the given fractions, and it will be the common denominator required: divide this common denominator by each particular denominator, and multiply the quotient by its own numerator for a new numerator; and these new numerators being placed over the common denominator, will express the fractions required, in the lowest terms.

1. **Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$ to their least common denominator.**

$$\text{Denominators are } \begin{array}{r|l} 4 & 4,\ 6,\ 8 \\ \hline 2 & 1,\ 6,\ 2 \\ \hline & 1,\ 3,\ 1 \end{array}$$

$4\times2\times3\times1=24$ Least Common Denominator.

$$\begin{array}{ll} \text{First Denominator} & 4 \\ \text{Second Denominator} & 6 \\ \text{Third Denominator} & 8 \end{array}\Bigg\}\ 24\ \Bigg\{\begin{array}{ll} 6\times3=18 & \text{1st. Numerator.} \\ 4\times5=20 & \text{2d. Numerator.} \\ 3\times7=21 & \text{3d. Numerator.} \end{array}$$

Divide the first by 6; 2d. by 4; and 3d. by 3.

Ans. $\frac{18}{24}$, $\frac{20}{24}$, $\frac{21}{24}$ | $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$ *Proof.*

2. **Reduce $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{9}$.**

$$\begin{array}{r|l} 3 & 3,\ 5,\ 9 \\ \hline & 1,\ 5,\ 3 \end{array}$$

$$\begin{array}{c} 3 \\ 5 \\ 9 \end{array}\Bigg\}\ 45\ \Bigg\{\begin{array}{l} 15\times2=30 \\ 9\times4=36 \\ 5\times5=25 \end{array} \qquad \frac{30}{45} \quad \frac{36}{45} \quad \frac{25}{45}\ \textit{Ans.}$$

3. **Reduce $4\frac{1}{3}$, $3\frac{1}{3}$, $2\frac{3}{4}$.**

$$4\tfrac{1}{3},\ 3\tfrac{1}{3},\ 2\tfrac{3}{4} = \begin{array}{r|lll} & 13, & 10, & 11 \\ \hline 3 & 3, & 3, & 4 \\ \hline & 1, & 1, & 4 \end{array}$$

$$\begin{array}{c} 3 \\ 3 \\ 4 \end{array}\Bigg\}\ 12\ \Bigg\{\begin{array}{l} 4\times13=52 \\ 4\times10=40 \\ 3\times11=33 \end{array} \qquad \frac{52}{12},\ \frac{40}{12},\ \frac{33}{12}\ \textit{Ans.}$$

4. **Reduce $\frac{3}{4}$ of $\frac{8}{1}$, $\frac{5}{6}$ of $3\frac{1}{2}$.**

$$\frac{5}{6} \text{ of } \frac{\begin{array}{c}3\frac{1}{2}\\ 2\end{array}}{\overline{7}} \over 2$$

$$\begin{array}{r|ll} & 24, & 35 \\ \hline 4 & 4, & 12 \\ \hline & 1, & 3 \end{array} \text{ least common denominator.}$$

$$=12\div4\times24=\frac{72}{12},\ \frac{35}{12}\ \textit{Ans.}$$

$$12\div12\times35=$$

5. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

$\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$ *Ans.*

CASE VII.

To reduce a fraction of one Denomination to a fraction of another, but greater Denomination, retaining the same value.

RULE.

Reduce the given fraction to a compound one, by comparing it with all the denominations, between that given, and that to which you would reduce it. Then reduce this compound fraction to a simple one. (By case V.)

1. Reduce $\frac{3}{4}$ of a penny to the fraction of a pound.

By comparing it, it becomes $\overset{d.}{\frac{3}{4}}$ of $\overset{s.}{\frac{1}{12}}$ of $\overset{£.}{\frac{1}{20}}$ of a pound.

Multiply the Numerators together, $3\times1\times1=3$ and the Denominators together, $4\times12\times20=960$ or $\frac{1}{320}$ *Ans.*

2. Reduce $\frac{3}{4}$ of a farthing to the fraction of a pound.

$\frac{3}{4}$ of $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}=\frac{3}{3840}=\frac{1}{1280}$ *Ans.*

3. Reduce $\frac{6}{7}$ of a shilling to the fraction of a pound.

$\frac{6}{140}$ or $\frac{3}{70}$ *Ans.*

4. Reduce $\frac{3}{4}$ of a penny to the fraction of a shilling.

$\frac{3}{48}$ or $\frac{1}{16}$ *Ans.*

5. Reduce $\frac{3}{5}$ of a pennyweight to the fraction of a pound Troy.

$\frac{3}{1200}$ or $\frac{1}{400}$ *Ans.*

6. Reduce $\frac{3}{5}$ of a pound to the fraction of a hundredweight.

$\frac{3}{560}$ *Ans.*

7. Reduce $\frac{3}{4}$ of an hour to the fraction of a week.—$\frac{1}{224}$ *Ans.*

8. Reduce $\frac{3}{4}$ of a pint to the fraction of a hogshead.

$\frac{1}{672}$ *Ans.*

9. Reduce $\frac{4}{5}$ of a pound to the fraction of a guinea.

$\frac{4}{5}$ of $\frac{20}{1}$ of $\frac{1}{21}=\frac{8}{14}|\frac{0}{0}|\frac{4}{7}$ *Ans.*

10. Reduce $7\frac{1}{3}$ furlongs to the fraction of a mile.

$\frac{22}{3}$ of $\frac{1}{8}=\frac{22}{24}$, or $\frac{11}{12}$ *Ans.*

11. Reduce $\frac{2}{3}$ of an English crown, at 6*s*. 8*d*., to the fraction of a guinea.

$\frac{2}{3}$ of $\frac{80}{1}$ of $\frac{1}{12}$ of $\frac{1}{21}=\frac{16}{168}|\frac{0}{0}$ $\frac{2}{21}$ *Ans*

Case VIII.

To reduce a fraction of one Denomination to a fraction of another, but less, retaining the same value.

Rule.

Invert the order of the last rule, in comparing the given sum with the order of its respective denominations: then multiply the numerators together, and also the denominators.

Note.—In reducing farthings to pounds; and less to greater denominations of any table of weights, measures, &c., in comparing the less with the greater, the several different denominations become the *denominators*; but in reducing from greater to less, the several denominations become *numerators*, from the very nature of the respective comparisons. This case and case second, prove each other.

1. Reduce $\frac{1}{320}$ of a pound to the fraction of a penny.

By comparing it, it becomes $\frac{1}{320}$ of $\frac{20}{1}$ of $\frac{12}{1}=\frac{240}{320}$

$\frac{240}{320}\div$ by $80=\frac{3}{4}$ *Ans.*

2. Reduce $\frac{3}{960}$ of a pound to the fraction of a penny

$\frac{3}{960}$ of $\frac{20}{1}$ of $\frac{12}{1}=\frac{720}{960}$, or $\frac{3}{4}$ *Ans.*

[illegible]

[illegible] pound

[illegible]

[illegible] *Ans.*

[illegible] pound.

[illegible] *Ans.*

[illegible]

[illegible] *Ans.*

[illegible]

[illegible]

[illegible] the

[illegible]

Rule.

Multiply the numerator by the parts of the next inferior denomination, and divide the product by the denominator; if any thing remain, multiply it by the next inferior denomination, and again divide by the denominator; and thus proceed to the lowest denomination, should there be any remainders; the several quotients will be the answer, or value of the given fraction.

Example.

1. Find the value of $\frac{602}{960}$ of a pound.

```
                  602×by 20s. the next denom. to pounds
                   20
                  ----
Denom. 960)12040(12
                 960
                ----
                2440
                1920
                ----
                 520×by 12 next denom.
                  12
                ----
          960)6240(6
               5760
               ----
                480×by 4 next denom.
                  4
               ----
          960)1920(2
               1920
               ----
```

Ans. £0 12*s.* 6*d.* 2*qr.*

2. Find the value of $\frac{147}{160}$ of a pound.

Ans. £0 18*s.* 4*d.* 2*qr*

3. Find the value of $\frac{4}{5}$ of a hundredweight.

Ans. 3*qr.* 5*lb.* 9*oz.* 9*dr.*

4. Find the value of $\frac{7}{8}$ of 4*s*. 6*d*.

$4\times12+6=54.$ $\frac{7}{8}$ of $\frac{54}{1}=\frac{378}{8}$

Proof. Add $\frac{1}{8}$ of $\frac{54}{1}$ | 6*d*. 3*qr*. Rem. $2\times4\div8.$

Rem. $6\times4\div8$

s.	*d.*	*qr.*
3	11	1
	6	3
4	6	0=4*s*. 6*d*. *Prf.*

d. qr. s. d. qr
378÷8=47 1=3 11 1 *Ans.*

5. Find the value of $\frac{77}{128}$ of a pound Avoirdupois.
9*oz*. 10*dr*. *Ans.*

6. Find the value of $\frac{4}{5}$ of a hogshead of wine.
50*gal*. 1*qt*. $1\frac{1}{5}$ *Ans.*

7. Find the value of $\frac{8}{9}$ of a dollar, at 6*s*. *Ans.* 5*s*. 4*d*.

8. Find the value of $\frac{6}{7}$ of a guinea. *Ans.* 18*s*.

9. Find the value of $\frac{2}{3}$ of a moidore. *Ans.* 18*s*.

10. Find the value of $\frac{6}{7}$ of £1 1*s*.

$21\times6\div7=18s.$ *Ans.*

11. Find the value of $\frac{6}{7}$ of £6 18*s*.—*Ans.* £5 18*s*. 3*d*. $1\frac{5}{7}$*qr*.

12. Find the value of $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{7}{8}$ of a hogshead of wine.
Ans. 20*gal*. 2*qt*. $1\frac{3}{8}$*pt*.

13. Find the value of $\frac{4}{5}$ of \$1. *Ans.* 80*cts*.

14. Find the value of $\frac{2}{5}$ of a weekly month.
Ans. 1*w*. 4*d*. 4*h*. 48*min*.

Case X.

To reduce any given quantity to a fraction of any greater Denomination of the same kind.

Rule.

Reduce the given quantity to the lowest denomination mentioned in the given sum, for a numerator; and reduce the in-

teger or whole number, to the same denomination for a denominator. Reduce the fractions to the lowest terms.

Examples.

1. Reduce 6*s*. 8*d*. to the fraction of a pound.

6*s*. 8*d*. is reduced to pence, its lowest denominator for a numerator. One pound is the integer here to be reduced to pence for the denominator.

Thus 6*s*. 8*d*. is equal to 80 pence.
One pound is equal to 240 do. $\frac{80}{240}=\frac{1}{3}$ of a £1.—*Ans.*

2. Reduce 16*lb*. 4*oz*. to the fraction of a hundredweight. [Hundredweight is the integer or whole number.]
$\frac{260}{1792}=\frac{65}{448}$ *cwt.*—*Ans.*

3. Reduce 15*s*. 9*d*. to the fraction of a pound. [Pound is the integer.] $\frac{189}{240}=\frac{63}{80}$ Value is 15*s*. 9*d*.—*Ans.*

4. Reduce 4½*d*. to the fraction of a shilling. $\frac{18}{48}\Big|\frac{3}{8}$ *s*.—*Ans.*

5. Reduce 18*s*. to the fraction of a guinea. $\frac{18}{28}=\frac{9}{14}$ *gui.*—*Ans.*

6. Reduce 9*s*. to the fraction of a moidore. $\frac{9}{36}=\frac{1}{4}$ *moi.*—*Ans.*

7. Reduce 6*s*. 8*d*. to the fraction of a pound $\frac{80}{240}\Big|\frac{1}{3}$ £.—*Ans.*

ADDITION OF

VULGAR FRACTIONS.

Rule.

Reduce compound fractions to simple ones, mixed numbers to improper fractions, and fractions of different integers, to those of the same; and all of them to a *common* denominator or *least* common multiple; (by Case VI., Rule II.;) then the sum of the numerators, written over the common denominator, will give the sum of the fractions required.

Example.

1. Add $5\frac{1}{3}$, $\frac{2}{3}$, and $\frac{3}{4}$ together.

$$\begin{array}{r} 5\frac{1}{3} \\ 3 \\ \hline 16 \end{array}$$

$5\frac{1}{3}$ reduced to improper frac. $\frac{16}{3}$

Least common denominator $3\times4=12$

$$\begin{array}{r|ccc} & \frac{16}{3} & \frac{2}{3} & \frac{3}{4} \\ 3 & 3 & 3 & 4 \\ \hline & 1 & 1 & 4 \end{array} \qquad \left.\begin{array}{r} 3 \\ 3 \\ 4 \end{array}\right\} 12 \left\{\begin{array}{l} 4\times16=64 \text{ 1st Numerator.} \\ 4\times\ 2=\ 8 \text{ 2d Numerator.} \\ 3\times\ 3=\ 9 \text{ 3d Numerator.} \end{array}\right.$$

$$\frac{81}{12}$$

$$\frac{64}{12}\ \frac{8}{12}\ \frac{9}{12} \qquad \begin{array}{l} 12)81(6\frac{3}{4} \textit{ Ans.} \\ \underline{\ \ \ \ 72} \\ \ \ \ \ \ \ 9 \end{array}$$

2. Add $\frac{2}{3}$ of $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$ together.

$$\frac{6}{12}=\frac{1}{2}\quad\frac{4}{5}\quad\frac{5}{6}\qquad\frac{30}{60}+\frac{48}{60}+\frac{50}{60}=\frac{128}{60}=2\tfrac{2}{15}\ \textit{Ans.}$$

3. Add $\frac{3}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{5}{9}$ together. $\frac{1031}{360}=2\frac{311}{360}$ *Ans.*

4 Add $\frac{3}{5}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{2}{3}$ together. $\frac{30}{15}$ or 2. *Ans.*

Note 1.—In adding mixed numbers, which are not compounded with other fractions; first find the sum of the whole numbers; after which, find the value of the fractions, which add to the whole numbers.

5. Add $6\frac{1}{3}$, $5\frac{2}{3}$, $7\frac{5}{6}$ together.

$$3)\underline{\frac{1}{3}\quad\frac{2}{3}\quad\frac{5}{6}}$$

$$1\quad 1\quad 2=6 \text{ least common denominator.}$$

$6\div3\times1=2$	$6)11(1\frac{5}{6}$	Whole num. are $6+5+7=18$.
$6\div3\times2=4$	6	18
$6\div6\times5=5$	5	Add $1\frac{5}{6}$
11	6	$19\frac{5}{6}$ *Ans.*

6. Add $4\frac{3}{5}$, $5\frac{5}{8}$, and 15 together.

$$\begin{array}{r} 24 \\ 1\frac{9}{40} \\ \hline 25\frac{9}{40} \end{array}\ \textit{Ans.}$$

7. Add 18, $3\frac{3}{4}$, $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{8}{9}$ together.

$$18+3=21$$

$$\frac{661}{540}=1\frac{121}{540}\qquad 22\frac{121}{540}\ \textit{Ans.}$$

Note 2.—To add fractions of money, weight, &c., reduce fractions of different integers to those of the same: or, what is preferable, find the value of each fraction by itself, and then add them together in their proper denominations.

8. Add $\frac{5}{8}$ of a shilling to $\frac{3}{8}$ of a pound.

[*First Method.*]

$\frac{5}{8}$ of $\frac{1\times5}{20\times160}$ $\frac{3}{8}$ 480 1st Numerator.
40 2d Numerator.

520 Sum.

$160\times8=1280$

520
20

1280)10400(8
10240

160
12

1280)1920(1
1280

640
4

1280)2560(2
2560

£0 8*s.* 1*d.* 2*qr.* *Ans.*

[*Second Method.*]

$\frac{5}{8}$ of $\frac{1}{20}=\frac{5}{160}$

5
20

100
12

160)1200(7
1120

80
4

160)320(2
320

$\frac{3}{8}=\frac{3}{20}$

8)60(7
56

4
12

8)48(6
48

	£.	s.	d.	qr.
$\frac{3}{8}$=	0	7	6	0
$\frac{5}{160}$=	0	0	7	2
		8	1	2 *Ans.*

9. Add $\frac{5}{6}s$. $\frac{4}{5}$£. $\frac{3}{4}d$. together. *Ans.* £0 16*s.* 10*d.* 3*qrs*

10. Add $\frac{4}{5}$ of a ton, $\frac{8}{9}$ of a hundredweight together.
Ans. 16*cwt.* 3*qr.* 15*lb.* 8*oz.* 14$\frac{2}{9}$*gr*

11. Add $\frac{7}{8}$ of a mile to $\frac{9}{10}$ of a furlong. *Ans.* 7*fur.* 36*rd*

12. Add $\frac{1}{2}w$. $\frac{2}{3}d$. $\frac{1}{3}h$. and $\frac{3}{4}m$. together.
Ans. 4*d.* 4*h.* 20*m.* 45″.

13. Add $\frac{3}{4}yd$. $\frac{1}{3}ft$. $\frac{5}{8}m$. together. *Ans.* 1100*yd.* 2*ft.* 7*in.*

SUBTRACTION OF

VULGAR FRACTIONS.

Rule.

Prepare the given fractions as in addition; then subtract the less numerator from the greater, and place the difference over the common denominator, and it will give the difference of the fractions required.

Note.—When the given fractions are of different denominations, reduce them to their proper values, and then subtract as in compound subtraction.

Example.

1. From $\frac{4}{7}$ take $\frac{7}{8}$.

$$\left.\begin{array}{l} 4\times8=32 \\ 7\times7=49 \end{array}\right\}\text{Numerators.} \qquad \text{Then } \left.\begin{array}{r} 49 \\ 32 \end{array}\right\}\text{Subtracted.}$$

$$7\times8=56 \text{ Denominator.} \qquad \frac{17}{56}\text{ difference.}$$

Ans. $\frac{17}{56}$.

2. From 14 take 6½.

$$\frac{14}{1}\quad\left.\frac{13}{2}\right\}\begin{array}{r} 28 \\ 13 \\ \hline 15 \end{array} \qquad 2)15(7\tfrac{1}{2}\ \textit{Ans.} \quad \begin{array}{r} 14 \\ \hline 1 \end{array}$$

3. From $13\frac{1}{3}$ take $\frac{3}{4}$ of 18.

$$\frac{40}{3} \quad \frac{54}{4} \qquad \begin{array}{r} 162 \\ 160 \\ \hline \end{array}$$

$$\therefore \frac{2}{12} \quad \textit{Ans.}\ \frac{1}{6}.$$

4. From $4\frac{1}{2}$ added to $4\frac{1}{2}$, take $\frac{3}{4}$ of 12. *Ans.* 0
5. From $\frac{7}{8}$ of 64, take $\frac{11}{12}$ of 22. *Ans.* $35\frac{5}{6}$.
6. From $\frac{2}{3}$ of a pound take $\frac{2}{3}$ of a shilling. *Ans.* 12*s*. 8*d*.
7. From $\frac{2}{3}$ of a shilling take $\frac{3}{4}$ of a penny. *Ans.* 7*d*. 1*qr*.
8. From $\frac{2}{3}$ of an ounce take $\frac{9}{10}$ of a pennyweight. *Ans.* 7*oz*. 2*pwt*.
9. From $\frac{5}{6}$ of $\frac{2}{3}$ of a league take $\frac{6}{7}$ of a mile. *Ans.* 6*fur*. 19*rd*. 0*ft*. 10*in*. $2\frac{2}{7}$*bc*
10. From $10\frac{1}{2}$ weeks take $16\frac{3}{8}$ days. *Ans.* 8*w*. 1*d*. 3*h*.

MULTIPLICATION OF

VULGAR FRACTIONS.

Rule.

Reduce compound fractions to simple ones, mixed numbers to improper fractions, and fractions of different integers to those of the same: then multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

Example

1. Multiply $\frac{5}{7}$ by $\frac{3}{9}$.

$$\begin{array}{l} 5\times3=15 \\ 7\times9=\overline{63} \end{array} \qquad \textit{Ans.}\ \frac{15}{63}$$

2. Multiply $6\frac{3}{4}$ by $4\frac{2}{3}$.

$$\frac{27}{4} \times \frac{14}{3} = \frac{378}{12} \qquad 12)378(31\tfrac{1}{2}\ \textit{Ans.}$$

```
12)378(31½ Ans.
   36
   --
    18
    12
    --
     6
```

3. Multiply $\frac{7}{8}$ of $\frac{5}{6}$, by $\frac{2}{3}$ of $\frac{5}{7}$. $\frac{350}{1008}=\frac{25}{72}$ *Ans.*
4. Multiply $8\frac{1}{5}$ by $\frac{2}{3}$ of $3\frac{1}{2}$. $\frac{574}{30}\div 30=19\frac{2}{13}$ *Ans.*
5. Multiply $\frac{3}{5}$ of $\frac{5}{6}$, $\frac{2}{3}$ of $4\frac{1}{2}$, and $\frac{9}{10}$ of $\frac{6}{7}$. $\frac{14580}{18000}=\frac{81}{100}$ *Ans.*
6. Multiply $\frac{2}{3}$ of 9, by $\frac{7}{8}$ of 5. $\frac{630}{24}=26\frac{1}{4}$ *Ans.*
7. Multiply $3\frac{2}{5}$ by $2\frac{2}{3}$, and that product by $\frac{1}{3}$ of $\frac{9}{10}$.
$\frac{7956}{1250}$ or $6\frac{228}{625}$. *Ans.*

DIVISION OF

VULGAR FRACTIONS.

Rule.

Prepare the fractions as in multiplication; then invert the divisor, and proceed exactly as in multiplication. The *product* will give the *quotient* required.

Example.

1. Divide $\frac{6}{7}$ by $\frac{4}{5}$.

$$\frac{6\times5=30}{7\times4=28}\div 30=1\tfrac{1}{14}.\ \textit{Ans.}$$

2. Divide $\frac{7}{8}$ by $\frac{8}{9}$ $\frac{63}{64}$ *Ans.*
3. Divide $11\frac{1}{4}$ by 7 [illegible] *Ans.* $1\frac{1}{2}$.
4. Divide 25 by $\frac{3}{8}$ $\frac{200}{3}$ or $66\frac{2}{3}$ *Ans.*

5. In proof of the last sum, multiply the quotient $\frac{200}{3}$ by the divisor $\frac{3}{8}$, and it leaves the dividend.

$$\frac{200}{3}\times\frac{3}{8}=\frac{600}{24}$$

```
24)600(25 Ans.
   48
   ---
   120
   120
   ---
```

6. Divide 67 by 113 $\frac{67}{113}$ *Ans.*
7. Divide 113 by 67. $\frac{113}{67}$ or $1\frac{46}{67}$ *Ans*

Note.—It must have been noticed, that the operations of Arithmetical rules, with whole numbers and fractions, produce contrary effects. Vulgar, as well as Decimal Fractions, claim a share in this seeming paradox. In this rule of the Division of Vulgar Fractions, it would seem formidable; yet perhaps, by a careful attention to the nature and effects produced by fractions, the veil may, in some degree, be removed.

The problem, already advanced, should be kept in mind; viz.—that when the numerator *only* of a fraction is increased, the value of the fraction becomes *greater*; but when the denominator *only* is increased, the value becomes *less*: and consequently, if the numerator and denominator are both equally increased, or equally diminished, the value in either case is not altered. Hence, if a given number be multiplied by one, it receives no alteration; if a number be multiplied by a whole number greater than one, its value is increased; if a given number be multiplied by a proper fraction, that is, taken for half, thirds, quarters, &c., the value of the given number is diminished, and the product, of course, is less than the multiplicand. With division the effects are different. The quotient, arising from the division of any given number, by a proper fraction, is *greater* than the dividend. *Example,*—4÷by $\frac{2}{5}$=10. It is hence evident, that a whole number contains more halves, thirds, quarters, &c. than it contains units: and consequently, if a whole number and a fraction be divided in the same manner, the quotients will bear the same proportions to the respective num-

bers divided. But, it has already been seen, that the value of a *whole* number is increased, when the divisor is a proper fraction; it follows, therefore, that the value of the fraction is also increased, in the same manner. *Example,*—$\frac{4}{5}$÷by $\frac{4}{6}$=$\frac{24}{20}$=1$\frac{1}{5}$.

More clearly to explain the effect of this operation, suppose $\frac{2}{5}$ were to be divided by $\frac{7}{9}$, the process would be thus; $\frac{2}{5}\times\frac{9}{7}$=$\frac{18}{35}$, quotients. If $\frac{2}{5}$ were to be divided by 7 only; or it was required to take one seventh part of the dividend, $\frac{2}{5}$; that is readily obtained by multiplying the denominator 5 by 7; because the value of a fraction is always *decreased* by increasing its denominator only; as $\frac{2}{5}$×by 7=$\frac{2}{35}$. Now, instead of two parts out of five parts, as before, it is but two parts out of thirty-five parts, the denominator 5 having been increased seven-fold. But if, instead of dividing by 7, it were $\frac{7}{9}$; as $\frac{7}{9}$ is nine times less than 7, (*Ex.* 63÷by $\frac{7}{9}$=441÷by 9=49: and $\frac{63}{1}\times\frac{7}{1}$=441: now the quotient of $\frac{7}{9}$, viz. 49 multiplied by 9, produces 441, equal to the quotient arising from dividing by 7, viz. 441, which was to be proved;) so the quotient of *any* number divided by $\frac{7}{9}$, will be 9 times *greater* than the quotient of the *same* number, divided by 7. *Ex.* 63÷$\frac{7}{9}$ will be 9 times greater than the quotient of 63÷7. Thus, $\frac{63}{1}$×9÷by 7, gives 81 quotient; but 63÷by 7, gives 9 quotient. This quotient 9×by 9=81, the first quotient. This therefore exhibits the necessity of multiplying the numerator 2, of the fraction $\frac{2}{5}$ by 9, to produce the quotient $\frac{18}{35}$. Hence the contrary effects of whole numbers and fractions.

Questions relative to Fractions; and more particularly to Vulgar Fractions.

1. What are fractions? and whence is the term derived?
2. Whence arise fractions? and why are they used?
3. How many kinds of fractions are there?
4. What is a Vulgar Fraction, and how is it expressed?
5. What is the meaning of the term Denominator? and what is its use?
6. What is the meaning of the term Numerator? and what is its use?

7. Are these terms arbitrary, or are they explanatory of their use?

8. When does a Vulgar Fraction express its greatest value? when the parts of the numerator most nearly equal the parts of the denominator, or the reverse?

9. If the parts of the numerator and denominator are increased by an equal multiplier, or decreased by an equal divisor, how is the value of the fraction effected?

10. If the parts of the numerator only be increased, what is the effect?

11. If the parts of the denominator only be increased, what is the effect?

12. Can a Vulgar Fraction be expressed in an almost endless variety of forms, and all of which have the same signification?

13. By what various names are Vulgar Fractions distinguished?

14. What is a single, simple, or proper fraction? and how are they distinguished?

15. What is an improper fraction?

16. What is a mixed number?

17. What is a compound fraction?

18. How may a whole number be expressed fraction-wise?

19. What is the common measure of two or more numbers?

20. What is the greatest common measure?

21. What is the common multiple?

22. When is a fraction said to be in its least or lowest terms?

23. How is the least common multiple found?

24. What is meant by the Reduction of Vulgar Fractions?

25. How are fractions reduced to their lowest terms?

26. Is the value of a fraction altered by being reduced to its lowest terms?

27. How may a mixed number be reduced to its equivalent improper fraction?

28. When a mixed number is reduced to its equivalent fraction, what species of a fraction does it invariably form?

29. How is an improper fraction reduced to its equivalent whole, or mixed numbers?

30. How may a whole number be reduced to an equivalent fraction, having a given denominator?

31. How may a compound fraction be reduced to an equivalent simple one?

32. If the denominator of one fraction equal the numerator of another, how may they be disposed of?

33. May numerators and denominators ever be contracted by taking aliquot parts?

34. How may fractions of different denominators be reduced to equivalent fractions, having a common denominator?

35. What is the reason of the last rule?

36. How are fractions reduced to the least common denominator?

37. Is any benefit derived by this rule, over that of finding a common denominator?

38. How is the fraction of one denomination reduced to the fraction of another, but greater, yet retaining the same value?

39. On what principle is this change accomplished?

40. How may a fraction of one denomination be reduced to the fraction of another, yet less, but retaining the same value?

41. How is this change produced?

42. When less denominations are changed into greater, what constitutes the several denominators used in comparing the given fraction?

43. When greater denominations are changed into less, what constitutes the several numerators used in comparing the given fraction?

44. How is the value of a vulgar fraction found; and why is it thus found?

45. How may any given quantity be reduced to a fraction of any greater denomination of the same kind?

46. What is meant by the integer, or whole number, required to be reduced?

47. What is the rule for the Addition of Vulgar Fractions?

48. How may mixed numbers be added, which are not compounded with other fractions?

49. What is the most convenient method of adding money, weights, or measures of different denominations?

50. What is the rule for the Subtraction of Vulgar Fractions ?

51. When the fractions are of different denominations, what is the most convenient method of subtracting ?

52. What is the rule for the Multiplication of Vulgar Fractions ?

53. What is the rule for the Division of Vulgar Fractions ?

54. What difference is there in preparing fractions for Addition and Subtraction, and for Multiplication and Division ?

55. What different effects result between the Division of whole numbers and that of Vulgar Fractions ?

DECIMAL FRACTIONS.

THE nature of Decimal Fractions has already been partially explained, under the head of Notation or Numeration. As they are founded entirely on the principle of tens, although in retrograde ratio from the place of units towards the right-hand, yet they are essentially subject to the general rules adopted for the regulations of whole numbers. The numerator only is clearly expressive of their true value ; and in no instance, perhaps, is it rendered imperious to express its denominator : it being at all times, if expressed, either 10, 100, 1000, &c. ; that is, unity or one, with one or more ciphers annexed to it ; it is always well known what it must necessarily be, from the number of places of figures occupied by the numerator. If but one place in decimals is used, the denominator would be tenths ; if two places, hundredths ; if three places, thousandths, &c. ; that is, the denominator, if expressed, would be the number of the term given to the right-hand figure in numerating them, the left-hand figure of the decimal being called tenths. If there are three places of decimals, the third, or right-hand figure is called thousandths, which would be the number of the denominator, if expressed. If four places of decimals, then it would be ten thousandths ; viz. four ciphers are annexed to unity in the denominator. They are a species of Fractions therefore, very different from

Vulgar Fractions, and far less tedious and intricate to learners. A great difficulty, in Vulgar Fractions, arises from the variety of denominators; for when numbers are divided and subdivided into different *kinds* of parts, they cannot so easily be compared. It was this which probably gave rise to the invention of Decimal Fractions, where the units were divided into *similar* parts; and the whole of the operations are regulated upon the general plan of whole numbers.

It will be carefully remembered, that Decimals are distinguished from whole numbers by having a point, generally denominated a separatrix, placed before them, or dividing them from whole numbers. Also, that ciphers placed at the left-hand of decimals, decrease the value in a ten-fold ratio, by removing the significant figure farther from the unit's place, or separatrix; but ciphers placed on the right-hand of decimals, do not alter their value at all. Furthermore, that the first or left-hand figure of a decimal, usually expresses the principal value of it; the second is diminished in a ten-fold ratio; and the third in like manner. That three places of figures in decimals, are all that are usually necessary to express the decimal value. That it is not the *number* of places which the decimal occupies, but the *greatness* of the significant left-hand figures of the decimal, which denotes the principal value.

ADDITION OF DECIMALS.

Rule.

1. Place the numbers, whether mixed or pure decimals, under each other, according to the value of their places.

2. Find their sum as in whole numbers, and point off as many places for decimals, as are equal to the greatest number of decimal parts in any of the given numbers.

Example.

1. Find the sum of 36,4735+456,74387+12,567+375,8543+65,375.

36,4735
456,74387
12,567
375,8543
65,375
947,01367

2. Find the sum of 94,7865+132,9472+128,13+5,99584+38,773+272,18.

94,7865
132,9472
128,13
5,99584
38,773
272,18
672,81254

3. Find the sum of $528,75+34,625+107,875+55,25.

$528,75
34,625
107,875
55,25
$726,500

Note.—It is observable, that Decimals and *Federal Money* are subject to the same law of notation and operation. For a dollar being the whole number, or money unit; and a dime being a tenth, a cent the hundredth, and a mill the thousandth part of a dollar; it follows, that any number of dollars, dimes, cents, and mills, is simply the expression of dollars, and decimal parts of a dollar. Thus, 9 dollars, 5 dimes, 6 cents, and 5 mills, are,=$9,56*cts.* 5*m.*, or $9,$\frac{565}{1000}$ of a dollar.

4. Find the sum of $167,42+78,695+99,45+211,725+49,625+150,9+28,84+,75+320,50.

$167,42
78,695
99,45
211,725
49,625
150,9
28,84
,75
320,50

$1107,905

5. Find the sum of $729,185+242,22+516,395+108,734+62,625+185,472+337,75.

$729,185
242,22
516,395
108,734
62,625
185,472
337,75

$2182,381

6. Add one-ten-thousandth part of a unit to $9,9999, and the whole sum will be $10.

$9,9999
1

$10,0000

SUBTRACTION OF DECIMALS.

Place the numbers according to their values; then subtract as in whole numbers, and point off the decimals as in Addition.

Examples.

1. From $827,375 take $733,50.

$827,375
733,50
$ 93,875

2. From $602,975 take $407,29.

$602,975
407,29
$195,685

3. From $733,23 take $247,3756.

$733,23
247,3756
$485,8544

4. From $73,81 take $19,825.

$73,81
19,825
$53,985

5. From $217,38 take $118,472.

$217,38
118,472
$ 98,908

6. From $1000 subtract one mill. *Ans.* $999,999.

7. From $20 subtract one cent. *Ans.* $19,99.

8. From $101 subtract one dollar and three cents. *Ans.* $99,97.

MULTIPLICATION OF DECIMALS.

Rule.

1. Whether they be mixed numbers or pure decimals, place the factors under each other, and multiply them as in whole numbers.

2. Point off so many figures from the right-hand of the product, as there are decimal places in *both* the factors; and if there be not so many places in the product, supply the deficiency by prefixing ciphers.

Examples.

1. Multiply \$5,375 by 8 mills.

5,375
,008

,043000

Ans. ,04*cts.* and 3*m.*

2. Multiply \$4,005 by \$1 and 5 mills.

4,005
1,005

20025
400500

\$4,025025

Ans. \$4 2½*cts.* &c.

3. Multiply ,25*cts.* by ,25*cts.*

,25
,25

125
50

Ans. ,0625, or 6¼*cts.*

4. Multiply ,75*cts.* by ,75*cts.*

,75
,75

375
525

Ans. ,5626, or 56*ct.* 2½*m.*

Note.—How is this! Twenty-five cents×by 25 cents, give only a sixpenny chunk, a yankee fourpence half-penny, or a Pennsylvania fivepenny bit! Strange indeed! Well, this subject must be investigated.

It will be remembered, that decimals, as well as Vulgar Fractions, are not whole numbers, but merely parts of such. If whole numbers were multiplied into whole numbers, they would produce whole numbers. But if a whole number is multiplied by *parts* of a whole number, the product cannot be a *whole*, but must be a *part*. As 1 dollar×by 50 cents, would give only 50 cents, viz. the half of the multiplicand; and that, because the multiplier is only the half of a dollar, or of one unit. If 2 dollars were×by 25 cents, the product would be 50 cents; viz. as 25 cents is but ¼ of a dollar, so one-fourth of the multiplicand 2 dollars, is 50 cents. And, if only *parts* of a whole are multiplied by parts, the product necessarily is but *parts* of a *part*. It is hence clearly seen, why there must be as *many* decimal places in the product as there are in *both* the factors.

For the decimal places in both the factors, are respectively involved into each other by multiplying, and the effect in every instance, would produce only *parts*. Hence in the total product, their result would be greater than the true value, if there were not as *many* places of decimals, as there are in *both* of the factors. From this view of the necessary *decrease*, occasioned by the multiplication of parts, instead of *increase*, as by whole numbers; it is obvious, that the product in the multiplication of parts, will always be proportioned to the quantity in the parts of the multiplier. If we multiply 2 dollars by 2 cents, the product is 4 cents; if 1 dollar is multiplied by 5 cents, the product is 5 cents; if by 9 dimes, the product is 9 dimes, or 90 cents: if the multiplier is one-quarter of a whole number, the product will be one-quarter; if a half, the product is half; if three-quarters, the product is three-quarters; and so of any other multiplier whatever.

It hence follows, that the same cause which produces a *decrease* by multiplying parts, operates the *reverse* in the Division of Decimals, and produces an *increase*, occasioned entirely by the nature of parts, as has been already exhibited. Twenty-five cents÷by 25 cents, would give a dollar quotient.

These proofs must, I apprehend, afford satisfactory evidence of the causes, why these apparent contrary effects are produced between decimals and whole numbers; and also, that these effects do not impede whole numbers and decimals from being combined together in the several operations of Arithmetical rules; and likewise of producing a true result.

5. Multiply $12,50 by ,125.

```
$12,50
  ,125
------
  6250
 15000
------
$1,56250
------
```

Ans. $1 56¼*cts.*

6. Multiply $5,000 by ,625

```
 $5,000
   ,625
-------
  25000
 10000
30000
--------
3,125000
--------
```

Ans. $3,12½*cts*

7. Multiply ,54375 by ,07346.

```
       ,54375
       ,07346
   ----------
       326250
      217500
     163125
    380625
 ------------
 ,0399438750
```

8. Multiply 52 hundredths of a mill by 12½ cents.

```
   ,00052
     ,125
  -------
      260
     104
     52
  -------
 ,00006500
```

9. What would 327¾ bushels of wheat amount to, at $1 37½cts. the bushel?

```
Bushels.  327,75
Price.     1,375
         -------
          163875
         229425
         98325
        32775
       ---------
       $450,65625
```

Ans. $450 65cts. 6¼m.

To multiply by 10, 100, 1000, &c., remove the separatrix so many places from the left towards the right-hand, as there are ciphers in the multiplier.

	Multiplied by		
Thus, - - - - ,375	10	makes	3,75
	100	do.	37,5
	1000	do.	375,

For ,375 × 100 makes 37,500, &c.

Note.—The multiplication of decimals may also be contracted. Write the unit's place of the multiplier under that figure of the multiplicand, whose place you would reserve in the product; and dispose of the rest of the figures in an inverted order to that in which they are usually placed. In multiplying, reject all the figures which are at the right-hand of the multiplying digit, and note down the products, so that their right-

[illegible] ; ob-
[illegible] what
[illegible], &c.,
[illegible] ; and

[illegible] de-

[illegible]

[illegible] ient
[illegible] the
[illegible] from
[illegible] de-
[illegible] se in

[illegible] de re-
[illegible] ent.
[illegible] se in

the dividend, annex as many ciphers to the dividend, as will balance the divisor, and the quotient will be whole numbers If, however, there be a remainder, more ciphers may be added to the dividend, which will produce decimals in the quotient, and consequently there will be a greater degree of exactness. If there are no decimals in the divisor, then the decimal places in the quotient must equal those in the dividend. This also may render it necessary to annex ciphers to the dividend.

1. Divide 650,566875 by 88,5125.

```
88,5125)650,566875(7,35 quotient.
        6195875
        -------
         3097937
         2655375
         -------
          4425625
          4425625
          -------
```

2. Divide 48,8627764 by 7,234. *Ans.* 6,7546.
3. Divide ,00809723970 by ,0345. *Ans.* ,2347026.
4. Divide ,679025 by 865. *Ans.* ,000785.
5. Divide 12 by 325. *Ans.* ,036923076+

Note.—The addition mark at the end of the quotient, denotes that the division might have been continued; or there was a remainder left.

6. Divide $346,1653 by $504,12. *Ans.* ,68667+
7. Divide $1 by 12 cents. *Ans.* $8,333+
8. Divide $2 by 12 cents. *Ans.* $16,6666+
9. Divide $4 by 12 cents. *Ans.* $33,3333+
10. Divide $8 by 12 cents. *Ans.* $66,6666+
11. Divide 56 cents by $1,12. *Ans.* ,5.
12. If 18¾ yards, or 18,75 yards, cost $25,78125; what costs one yard? *Ans.* $1,37½.

Note.—When either decimals or whole numbers are to be divided by 10, 100, 1000, &c. viz. unity with ciphers, it is accomplished by removing the separatrix in the dividend, so

many places towards the left-hand as there are ciphers in the divisor.

695 divided by	10=quotient		69,5
	100=	do.	6,95
	1000=	do.	,695.

REDUCTION OF DECIMALS.

Case I.

To reduce a Vulgar Fraction to its equivalent Decimal.

Rule.

Annex ciphers to the numerator, and divide by the denominator; and the quotient will be the decimal required.

Note.—As many ciphers as are annexed to the given numerator, so many places must be pointed off in the quotient. If there be not so many places of figures in the quotient, make up the deficiency by prefixing ciphers.

Examples.

1. Reduce $\frac{1}{8}$ to a decimal.

```
8)1,000(,125 Ans.
  8
  --
  20
  16
  --
   40
   40
   --
```

2. Reduce $\frac{3}{4}$ to a decimal. *Ans.* ,75.

3. Reduce $\frac{1}{4}$ to a decimal. *Ans.* ,25

4. Reduce $\frac{1}{5}$ to a decimal. *Ans.* ,2.
5. Reduce $\frac{1}{2}$ to a decimal. *Ans.* ,5.
6. Reduce $\frac{4}{5}$ to a decimal. *Ans.* ,8.
7. Reduce $\frac{7}{8}$ to a decimal. *Ans.* ,875.
8. Reduce $\frac{6}{7}$ to a decimal. *Ans.* ,85714+
9. Reduce $\frac{1}{3}$ to a decimal. *Ans.* ,3333+
10. Reduce $\frac{2}{3}$ to a decimal. *Ans.* ,6666+
11. Reduce $\frac{3}{25}$ to a decimal. *Ans.* ,12.
12. Reduce $\frac{5}{174}$ to a decimal. *Ans.* ,2873+
13. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ to a decimal. *Ans.* ,375.

Case II.

To reduce quantities of several denominations to a decimal.

Rule.

1. Reduce the given quantity to the lowest denomination mentioned in the given sum, for a numerator; and reduce the integral part (or whole number,) to the same term for a denominator. Then reduce this Vulgar Fraction to its equivalent decimal: or, what I deem far better;

Rule 2.—Place the several denominations above each other, in regular gradations, having the highest denomination at the bottom. Then beginning at the top, divide each denomination by its value in the next greater denomination, and place the several respective quotients at the right-hand of the next superior denomination, as a decimal: thus continuing the divisions through all the denominations, and the last quotient will give the decimal required.

Examples.

1. Reduce 14*s.* 6*d.* 3*qr.* to tne decimal of a *pound.*

```
s. d. qrs.
14 6 3
12
——
174
  4
——
699qrs.
———
960
```

```
  1  Pound is the in-
 20  teger to be re-
 ——  duced to far-
 20  things, for a de-
 12  nominator.
 ——
240
  4
 ——
960qrs.
```

Divide the Numerator by the Denominator.

```
960)699,000000(,728125 Ans.
    6720
    ----
     2700
     1920
     ----
      7800
      7680
      ----
       1200
        960
       ----
       2400
       1920
       ----
        4800
        4800
        ----
```

Second Rule.

```
 4 | 3,00
12 | 6,7500
20 | 14,562500
   |   ,728125 Ans.
```

2. Reduce 13*s*. 4*d*. to the decimal of a pound.—*Ans*. ,666+
3. Reduce 6*s*. 8*d*. to the decimal of a pound.—*Ans*. ,3333+
4. Reduce 3*d*. 3*qr*. to the decimal of a shilling.—*Ans*. ,3125.
5. Reduce 14*s*. to the decimal of a pound. *Ans*. ,7.
6. Reduce 3*s*. 4*d*. New-England currency, to the decimal of a dollar. *Ans* ,55555+
7. Reduce 4*s*. 6*d*. New-York currency, to the decimal of a dollar. *Ans*. ,5625.

Note.—When the shillings are even, half the number, with a point prefixed, is their decimal expression. As 20 shillings make a pound, and pounds are governed by the principle of tens, $\frac{2}{10}$ are equal to $\frac{4}{20}$. But if the number of shillings be odd, annex a cipher to the shillings, and then halving them will give the decimal expression. It will be noticed, when the shillings are odd, a 5 falls into the second place of decimals; as 9=,45; as $\frac{1}{10}=\frac{2}{20}$, so 5 in the second place of decimals is the half of ,1 in the first place of decimals; and as $\frac{1}{10}$ is the decimal of 2 shillings, so 5 in the second place is precisely the half of ,1 in the first, and necessarily denotes one shilling.

8. Reduce the following shillings to decimals, viz. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19.

1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12,	13,	14,	15,
,05	,1	,15	,2	,25	,3	,35	,4	,45	,5	,55	,6	,65	,7	,75

16,	17,	18,	19,
,8	,85	,9	,95.

Thus 1,0, 3,0, 5,0 7,0 &c.
,05 ,15 ,25, ,35

9. What is the decimal expression of £9 17*s.* 6½*d.*
Ans. £9,877+

10. Reduce £43 13*s.* 4*d.* to a decimal expression.
Ans. £43,6666+

11. Reduce 3*qr.* 3*n.* to the decimal of a yard.—*Ans.* ,9375.

12. Reduce 16*gal.* 3*qt.* to the decimal of a hogshead.
Ans. ,26587.

13. Reduce 9*oz.* 15*pwt.* 20*gr.* to the decimal of a pound Troy. *Ans.* ,81597+

14. Reduce 3*cwt.* 3*qr.* 7*lb.* to the decimal of a hundred-weight. *Ans.* 3,8125.

15. Reduce 6*fur.* 30*pol.* to the decimal of a mile.
Ans. ,84375.

16. Reduce 4¾ calendar months, to the decimal of a year.
Ans. ,395833+

Case III.

To find the value of a decimal in the known parts of the integer.

Rule.

1. Multiply the given decimal by the number of parts in the next inferior denomination, and cut off as many places for a remainder from the right-hand of the product, as there are places in the given decimal.

2. Multiply the remainder by the next inferior denomination, and cut off a remainder as before; and thus proceed through

all the parts of the integer; and the several denominations standing on the left-hand of the separatrixes, exhibit the true answer.

Examples.

1. What is the value of ,6625 of a pound?

```
   ,6625
      20
  ------
 13,2500
      12
  ------
  3,0000   Ans. 13s. 3d.
  ------
```

2. What is the value of ,379 of a pound?—*Ans.* 7*s.* 6*d.* $3\frac{84}{100}$*qr.*
3. What is the value of ,875 of a pound? *Ans.* 17*s.* 6*d.*
4. What is the value of ,875 of a shilling? *Ans.* 10*d.* 2*qr.*
5. What is the value of ,9375 of a yard? *Ans.* 3*qr.* 3*n.*
6. What is the value of ,26587 of a hogshead?—*Ans.* 16*gal.* 3*qt.*
7. What is the value of ,8125 of a shilling? *Ans.* 9*d.* 3*qr.*
8. What is the value of ,8375 of an acre? *Ans.* 3*rd.* 14*p.*
9. What is the value of ,5375 of a year of 365 days?
   Ans. 196*d.* 4*h.* 30*m.*
10. What is the value of ,452 of a mile?
    Ans. 3*fur.* 24*rd.* 3*yd.* 1*ft.* 6,72*in.*
11. What is the value of ,9765 of a pound Troy?
    Ans. 11*oz.* 14*pwt.* 8,64*dr.*
12. What is the value of ,9554 of a pound?
    Ans. 19*s.* 1*d.* 1,184*qr.*

CONTRACTIONS OF DECIMALS.

Problem I.

A concise and ready method to find the decimal of any number of shillings, pence, and farthings, (to three places of figures,) by *inspection.*

Rule.

1. Write half the greatest even number of shillings for the first decimal figure. If the shillings be odd, add 5 in the second place of decimals.

2. Let the farthings in the given pence and farthings possess the second and third places of decimals; observing to increase the third place ¼ for every 6 farthings, occupying the second and third places; viz. for 6*qr.*, add ,25; for 12, add ,5; for 18, add ,75; for 24, add 1; for 30, add 1,25; and for 36, add 1,5.

Note.—The rule generally laid down, of increasing 1 for 12, and 2 for 36, is not accurate; nor will it correspond precisely with the answers obtained by the last rule. Recollect, that it requires 6¼ cents to make 6 pence, (New-York currency;) 12½ cents to make 1 shilling; 18¾ cents to make 1 shilling and 6 pence; 25 cents to make 2 shillings; 31¼ cents to make 2 shillings and 6 pence; and 37½ cents to make 3 shillings. This principle should regulate the several additions by *inspection*. By adopting this, if either of the above data were used, there would be no deviation in fractional parts from obtaining the precise value, as by the last rule; and should the number of farthings fall between the sixes, add for the nearest 6, and the fractional variation is of no value.

Examples.

1. Find the decimal of a pound of 9*s.* 9*d.* by inspection.

			[*Proof by Case III.*]
	s. d.		
	9 9		
	4	½ the num. of shill.	,4875
	5	for the odd shill.	20
	36	farthings in 9 pen.	
	15	excess of 36.	9,7500
			12
Ans.	,4875	Amount.	
			9,0000 *Ans.* 9*s.* 9*d.*

2. Find the decimal of a pound of 11*s.* 10*d.* 2*qr.* by inspection. *Ans.* ,59375.

3. Find the decimal of a pound of 14*s.* 4*d.* 2*qr.* by inspection. *Ans.* ,71875.

4. Find the decimal of a pound of 17*s.* 3*d.* by inspection. *Ans.* ,8625

5. Find the decimal of a pound of 1*s.* 6*d.* by inspection. *Ans.* ,075.

6. Find the decimal of a pound of 4*d.* 2*qr.* by inspection. *Ans.* ,01875

7. Find the decimal of a pound of £22 19*s.* 10*d.* 2*qr.* by inspection. *Ans.* £22,99375

8. Find the decimal of a pound of £3 2*s.* 9*d.* by inspection. *Ans.* £3,1375.

9. Find the decimals of 16*s.* 4*d.* 2*qr.*+15*s.* 6*d.*+2*s.* 7*d.* 2*qr.* +12*s.* 3*d.*+7*s.* 6*d.* by inspection, and also amount.

```
 ,81875
 ,775
 ,13125
 ,6125
 ,375
 -------
2,71250
     20
 -------
14,25000
     12
 -------
 3,00000
 -------  Ans. £2 14s. 3d.
```

PROBLEM II.

A short and easy method to find the value of any decimal of a pound by *inspection.*

RULE.

Double the first figure, or place of tenths for shillings; and if the second figure be 5, or more than 5, add another shilling; then having deducted 5 from the second figure of decimals, call the remaining figures in the second and third places so many farthings, abating 1,75 from 43,75; 1,5 from 37½; 1,25 from

31¼; 1 from 25; ,75 from 18¾; ,5 from 12½; and ,25 from 6¼. This rule is directly the reverse of the last.

Note.—When the given decimal has but two figures, if any thing remain after shillings are taken out, annex a cipher to it, or suppose it to be done.

Examples.

1. Find the value of ,4875 of a pound by inspection.

s.	*d.*	
8		double of first figure for shillings.
1		for second figure, it being over 5, deduct 5 from 8, and it leaves 375; from which deduct 1,75,
	9	leaves 36 farthings, viz. 9*d.*
9	9	*Ans.*

2. Find the value of ,59375 of a pound by inspection. *Ans.* 11*s.* 10*d.* 2*qr.*

3. Find the value of ,71875 of a pound by inspection. *Ans.* 14*s.* 4*d.* 2*qr.*

4. Find the value of ,8625 of a pound by inspection. *Ans.* 17*s.* 3*d.*

5. Find the value of ,075 of a pound by inspection.—*Ans.* 1*s.* 6*d.*

6. Find the value of ,01875 of a pound by inspection. *Ans.* 4*d.* 2*qr.*

7. Find the value of £22,99375 of a pound by inspection. *Ans.* £22 19*s.* 10*d.* 2*qr.*

8. Find the value of £3,1375 of a pound by inspection. *Ans.* £3 2*s.* 9*d.*

9. Find the value of ,81875+,775+,13125+,6125+,375= £2,71250. *Ans.* £2 14*s.* 0*d.* 3*qr*

Questions relative to Decimal Fractions.

1. On what principle are Decimals constituted?

How are they distinguished, and how do they differ from whole numbers?

3. Are they subject to the same rules of operation as whole numbers?

4. Is their value expressed by the Numerator, or Denominator?

5. Is it necessary their denominator be used at all in expressing clearly their value?

6. What would invariably be their denominator, if used, and how is it known?

7. How do Decimals differ from Vulgar Fractions?

8. Which is less tedious and intricate for learners?

9. What gave rise to the invention of Decimal Fractions?

10. What effect do ciphers have when prefixed to decimals, and also when annexed; and why such effects?

11. How many places of decimals are usually sufficient to express their chief value?

12. Which figure of the decimal denotes their principal value?

13. What is the rule for the Addition of Decimals? and also the proof?

14. Are Decimals and Federal Money subject to the same rules of operation?

15. Whence originated the system of computation in Federal Money?

16. What is the rule for the Subtraction of Decimals, and also its proof?

17. What is the rule for the Multiplication of Decimals, and also its proof?

18. Does the Multiplication of Decimals and whole numbers produce the same values?

19. Wherein do they differ; and why?

20. What is the rule for the Division of Decimals? and also its proof?

21. How does the Division of Decimals and whole numbers differ in the value of their respective quotients? and why?

22. Are the various minor rules in the multiplication and division of whole numbers, applicable generally to Decimals?

23. How is a Vulgar Fraction reduced to an equivalent decimal?

24. How are quantities of several denominations reduced to decimals?

25. Why does writing half the even number of shillings with a separatrix before it, give the decimal expression of a pound?

26. Why is 5 added in the second place of decimals, when the number of shillings is odd?

27. How is the value of a decimal found, in the known parts of the integer?

28. How is the decimal of a pound of any given number of shillings, pence, and farthings found by *inspection?* and why?

29. Why add ½ for 12; 1 for 24; and 1½ for 36, &c.?

30. How is the *value* of a decimal of a pound found by *inspection?*

31. Why abate ½ for 12½; 1 for 25, &c.?

REDUCTION OF CURRENCIES.

Rules for reducing the several currencies in the United States to Federal Money.

In the first settlement of these United States, a pound was of the same sterling value in all the Colonies as that of Great Britain; and a Spanish dollar was valued at 4*s*. 6*d*. In consequence of the legislatures of different colonies emitting bills of credit, which afterwards depreciated in their value, in some States more, and in others less; the currencies of the different States became regulated in the following manner:

New-England, Virginia, Ohio, Kentucky, Tennessee	6*s*. to a dol.	New-Jersey, Pennsylvania, Delaware and Maryland	7*s*. 6*d*. to a dollar.
New-York and North Carolina	8*s*. to a dol.	South Carolina and Georgia	4*s*. 8*d*. do.
Canada and Nova-Scotia	5*s*. to a dol.	Sterling Money	4*s*. 6*d*. do.

Note.—The reduction of the currencies may be greatly facilitated, by attending with a little care to the nature and neces-

sary construction of the several rules by which these different changes are effected. No rules are without a distinct meaning. They always arise from the nature and necessity of the case. This consideration should at all times influence the learner to enter at once into the nature and reasons of rules. The object is solely to exhibit the nature and principles of any rule, that examples are placed under it to exemplify the rule itself. They are not examples which will perhaps ever occur in any business transaction. Hence it is *ideas*, and not any *phraseology of words*, which should influence and direct the student. As well might a parrot be learned to talk, when influenced only by imitation, and totally destitute of any reasoning faculty, as a youth to acquire the least knowledge of figures, who never attended to the nature of a rule; and what is more singular, was never even required to repeat any rule. By understanding the principles on which rules are framed, the impressions become deeply riveted upon the mind, from a distinct comprehension of the *why's* and *wherefore's* they must be so; and such impressions will be permanent; while a treacherous memory, which depended entirely on the *shape of words*, without any reference to their meaning, "will vanish like the morning cloud and the early dew."

The more readily to acquire a clear knowledge of the reduction of currencies, make the rules by inspection, relative to Decimal Fractions, very familiar; and carefully attend to the nature of the given rules, with their accompanying demonstrations, why they *must* be so, and not otherwise; and a very short time will be amply sufficient for one to become a thorough proficient in this branch of Arithmetic. Indeed, the same would be true of every other branch of figures.

Case I.

In the reduction of the currencies of the different States to Federal Money, when the dollar is an even number of shillings, it is suitable to adopt the following

Rule.

When the sum consists of pounds only, annex a cipher to the pounds, and divide by half the number of shillings in a dollar, and the quotient will be dollars. Or, if there is a remainder after dollars, by annexing ciphers to the dividend, and continuing the division, cents, and perhaps mills, will be obtained.

Note.—The reason of this rule is clear. As pounds are managed on the principle of tens or whole numbers, and it requires two tens to make 20, the number of shillings in a pound; six-twentieths is the same in value as three-tenths. Thus adding a cipher to the pounds, is the same as multiplying by 10, and bringing them into tenths of a pound: and because a dollar is just three-tenths of a pound, New-England currency, dividing these tenths by 3, brings them into dollars, &c. Example.—One pound is $\frac{20}{1}$, and a dollar is $\frac{6}{1}$; so that a pound is to a dollar, as 20 is to 6; or it is $\frac{20}{6}$, which is equal to $\frac{10}{3}$. It follows, therefore, that multiplying by 10, and dividing by 3, will change pounds into dollars, or dollars and cents. If the give pounds were New-York currency, viz. 8 shillings to the dollar, the ratio then is $\frac{10}{4}$=to $\frac{20}{8}$; that is, add the cipher and divide by 4, and the quotient is Federal Money.

Rule II.

If the given sum consists of pounds, shillings, pence, &c. reduce the fractional part to the decimal of a pound by inspection, and annex the same decimally to the pounds, and divide by half the number of shillings in a dollar; and from the right-hand of the quotient, cut off *one figure less* for decimals, than there are decimal places in the dividend. Or if you divide New-England currency by ,3-tenths; or New-York currency by ,4-tenths, the ciphers must necessarily be annexed to the pounds, to balance the decimal in the divisor, and the quotient will be the same as before.

Note.—The reasons are the same in this second rule, as in the former. The principle is not altered because there are de-

cimals; for being the decimal of a pound, they bear their relative proportions equally the same to the fractional parts of a dollar.

Examples.

Reduce £375, New-England, Virginia, &c. currency, to Federal Money.

3)3750

$1250 *Ans.*

Reduce £1000 of N. E. do.

3)10000,000

$ 3333,333 *Ans.*

Reduce £462 New-England, &c. to Federal Money.

3)4620

$1540 *Ans.*

Reduce £1200, New-York and North-Carolina currency to Federal Money.

4)12000

$ 3000 *Ans*

Reduce £529 New-York & North-Carolina, &c. to Federal Money.

4)5290,0

$1322,5 *Ans.*

Reduce £213 12*s.* 4*d.* 2*qr.* New-England and Virginia currency to Federal Money.

By 2d rule. 3)213,61875

$ 712,0625 *Ans.*

Re. £450 18*s.* 9*d.* N. E. &c. to Fed. Mo.

3)450,9375

$1503,125 *Ans.*

Reduce £100 16*s.* 6*d.* New-York and North Carolina currency to Federal Money.

4)100,82500		£ *s.* *d.* *qr.*
$ 252,0625 *Ans.*	Reduce	530 18 6 2 New-York and North Carolina, &c. to &c.
		4)530,92700
		$1327,3175

Case II.

To reduce the currency of New-Jersey, Pennsylvania, Delaware and Maryland to Federal Money.

Rule.

Multiply the given sum by 8, and divide the product by 3, and the quotient will be dollars, &c. Should there be shillings, pence, &c. in the given sum; reduce them by inspection to the decimal of a pound, then multiply and divide as above.

Note.—The reason of multiplying by 8, and dividing by 3, is this:—the currencies of the above named States are 7*s.* 6*d.* to the dollar; viz. 90 pence. The pence in one pound 240. It stands thus, $\frac{240}{90}$, reduced to the lowest terms, make $\frac{8}{3}$. So that a pound is proportioned to a dollar, as 8 to 3.

Examples.

Reduce £264, New-Jersey, &c. currency to Federal Money.

$264 \times 8 = 2112 \div 3 = \$704$ *Ans.*

Reduce £125 8*s.* New-Jersey, &c. currency to Federal Money.

$125,4 \times 8 = 10032 \div 3 = \$334,40$ *Ans.*

Reduce £110 8*s.* 6*d.* New-Jersey, &c. currency to Federal Money.

$110,425 \times 8 = 883400 \div 3 = \$294,466$ *Ans.*

Case III.

To reduce the currency of South Carolina and Georgia to Federal Money.

Rule.

Multiply the given sum by 30, and divide the product by 7, and the quotient will be dollars, cents, &c.

Note.—A dollar of this currency is 4*s*. 8*d*. or 56 pence. It is then $\frac{240}{56}$, which, divided by 8, gives $\frac{30}{7}$. Hence it must be multiplied by 30, and divided by 7, to change their currency to Federal Money.

Examples.

Reduce £200, South Carolina, &c. currency to Federal Money.

$200\times30=6000\div7=\$857,142$ *Ans.*

Reduce £87 14*s*. 6*d*. South Carolina, &c. currency to Federal Money.

$87,725\times30=2631750\div7=\$375,964$ *Ans.*

Case IV.

To reduce the currency of Canada and Nova Scotia to Federal Money.

Rule.

Multiply the given sum by 4, and the product will be dollars.

Note.—Five shillings of this currency are equal to a dollar; of course 4 dollars make a pound; as 5 shillings $=\frac{240}{60}=\frac{4}{1}$. It is therefore to multiply only by four.

Examples.

Reduce £250, Canada and Nova Scotia currency, to Federal Money.

$250\times4=\$1000$ *Ans.*

Reduce £75 10*s*. 6*d*. &c. to Federal Money.

$75,525\times4=\$302,10$ *Ans.*

Case V.

To reduce Sterling to Federal Money.

Rule.

Multiply the given sum by 40, and divide that product by 9, and the quotient will be dollars, cents, &c.

Note.—Four shillings and sixpence is a dollar sterling Therefore $\frac{240}{54}$ is equal to $\frac{40}{9}$.

Examples.

Reduce £100 Sterling currency to Federal Money.

100×40=4000÷9=$444,444 *Ans.*

Reduce £100 18*s.* 3*d.* Sterling currency to Federal Money.

100,9125×40÷9=$448,50 *Ans.*

REDUCTION OF COINS.

Rules

For reducing Federal Money to the several currencies of the different States in the Union, also to Canada and Nova Scotia, Sterling, &c. are directly the *reverse* of those already given to bring the several currencies into Federal Money. It is sufficient merely to change the places of the multiplier and divisor, in their correspondent rules, and then proceed as in the former rules, and Federal Money will be again restored to the several currencies from which they were reduced.

Thus, in all the States whose currency is 6*s.* to the dollar, multiply the given dollars, cents, &c. by ,3 decimal; and the

N

product will be pounds, and decimal parts of a pound. The value of the decimal is easily found by inspection.

In the currency at 8*s*. to the dollar, multiply the dollars and cents by ,4 decimal; and the product will be as in the last rule, viz. pounds and decimals of a pound

In the currency of 7*s*. 6*d*. to the dollar, multiply by 3, and divide the product by 8; and the quotient will be as in the two last rules, pounds and decimals.

In the currency of 4*s*. 8*d*. to the dollar, multiply by 7, and divide by 30, and the answer will be as above.

In the currency of 5*s*. to the dollar, divide the given sum by 4, and the quotient will be pounds, and decimals of a pound.

In Sterling currency, at 4*s*. 6*d*. to the dollar, multiply by 9, and divide by 40, and the quotient is pounds, and decimals of a pound.

Examples

To illustrate the foregoing rules.

1. Reduce $1250 to New-England, &c. currency.

1250×,3=375,0—*Ans*. £375.

2. Reduce $712,0625, viz. 6¼ cents, to New-England, &c. currency.

712,0625×,3=£213,61875—*Ans*. £213 12*s*. 4*d*. 2*qr*.

3. Reduce $3000 to New-York and North Carolina, &c. currency.

3000×,4=£1200,0—*Ans*. £1200.

4. Reduce $252,0625, viz. 6¼ cents, to New-York and North Carolina currency.

252,0625×,4=£100,82500—*Ans*. £100 16*s*. 6*d*.

5. Reduce $704 to New-Jersey, &c. currency.

704×3=2112÷8=264—*Ans*. £264.

6. Reduce $294,4666 to New-Jersey, &c. currency.

294,4666×3=883,4000÷8=110,4250—*Ans*. 110 8*s*. 6*d*.

7. Reduce $857 14*ct.* 2*m.*+ to South Carolina and Georgia currency.

857,142×7=6000,000÷30=200—*Ans.* £200.

8. Reduce $375,964 to South Carolina and Georgia currency.

375,964×7=2631,748÷30=87,725—*Ans.* £87 14*s.* 6*d.*

9. Reduce $302 10*cts.* to Canada and Nova Scotia currency

302,100÷4=75,525—*Ans.* £75 10*s.* 6*d.*

10. Reduce $1000 to Canada and Nova Scotia currency.

1000÷4=250—*Ans.* £250.

11. Reduce $444,444+ to Sterling currency.

444,444×9=4000,000÷40=100—*Ans.* £100.

12. Reduce $448 50*cts.* to Sterling currency.
Ans. £100 18*s.* 3*d.*

To turn any given number of shillings, pence, and farthings, of any currency, into cents and mills.

RULE.

Reduce the given shillings and pence into pence; and if there are farthings, reduce them to the decimal of a penny, and annex the decimal to the pence, which becomes the dividend. Divide this by the number of pence in a dollar, of whatever currency is desired, and point off in the quotient, as the division of decimals directs; and if the given pence were less than a dollar, the quotient will be cents, mills, and parts of mills.

Note.—If the given sum exceed a dollar; then the figures on the left of the separatrix will be dollars; and those on the right would be cents and mills.

Examples.

1. Reduce 3*s*. 6*d*. 2*qr*. to cents and mills.

Pence in a dollar, New-England, &c. currency — *ct. m. pt.* 72)42,50000(,59 02 7+ *Ans.*

New-Jersey, &c. cur. — *ct. m. pt.* 90)42,5000(,47 2 2 *Ans.*

New-York, &c. cur. — *ct. m. pt.* 96)42,5000(,44 2 7 *Ans.*

South Carolina & Geor. currency — *ct. m. pt.* 56)42,5000(,75 8 9 *Ans.*

2. Reduce 2*s*. 11*d*. 3*qr*. to cents and mills.

Pence in a dollar, New-England, &c. currency — *ct. m. pt.* 72)35,7500(,49 6 5 *Ans.*

New-York & North Carolina currency — *ct. m. pt.* 96)35,7500(,37 2 3 *Ans.*

New-Jersey, &c. cur. — *ct. m. pt.* 90)35,7500(,39 7 2 *Ans.*

South Carolina & Geor. currency — *ct. m. pt.* 56)35,7500(,63 8 3 *Ans.*

To turn cents and mills into shillings, pence, and farthings.

RULE.

Multiply the cents and mills by the number of shillings in a dollar. Cut off from the right-hand of the product, as in the Multiplication of Decimals. Multiply the remainder by 12, the pence in a shilling: again, cut off as before; and if any remainder, multiply it by 4 farthings. The quotients on the left-hand of the separatrixes, will be shillings, pence, &c.

1. Reduce ,625 to New-England currency.

```
  ,625
     6
  ----
 3)750
    12
  ----
 9)000
  ----
```

Ans. 3*s*. 9*d*.

2. Reduce ,625 to New-York currency.

```
  ,625
     8
  ----
 5)000
  ----
```

Ans. 5.

3. Reduce ,625 to New-Jersey currency.

```
      ,625
        7½
      ----
      3125
     4375
     -----
    4)6875
        12
     -----
   8,)2500
         4
     -----
    1)0000    Ans. 4s. 8d. 1qr.
     -----
```

4. Reduce ,625 to South Carolina and Georgia currency.
Ans. 2*s.* 10*d.* 3*qr.*

REDUCTION OF CURRENCIES.

In *changing* the currency of one State into that of another of a different currency, also Canada, Nova Scotia, sterling, &c.; it is important for the learner to become familiar with the principles on which the rules are founded. *Ten minutes* of strict attention to this subject, will probably render it plain and permanent in the mind. There is *one* general principle that may be adopted, to frame the requisite rules for *all* the several *changes* of currencies. It is this:—

Take the number of pence in a dollar of the given currency to be changed for a denominator; and also the number of pence in a dollar of the currency into which it is to be changed for a numerator. Reduce this Vulgar Fraction to its lowest terms; then multiply the given sum by the numerator, and divide that product by the denominator, and the answer is obtained in the currency sought.

By changing the numerator and denominator, and proceeding as above, the currency is restored to its former state.

[illegible] used
[illegible] into
[illegible] anged
[illegible] n the
[illegible] so as
[illegible] ready

[illegible] ns, is,
[illegible] on of
[illegible] comes
[illegible] g the
[illegible] some
[illegible] s, for
[illegible] e ob-
[illegible] num-
[illegible] hence
[illegible], and
[illegible]

Examples.

[illegible]

4. Reduce £133 6s. 8d. New-England, &c. to sterling.

Fraction is $\frac{54}{72}=\frac{3}{4}$ £133 6s. 8d.
3

÷4=£100 Ster. *Ans.*

5. Reduce £150 New-York, &c. currency to New-England, &c.

$\frac{72}{96}=\frac{3}{4}$ *Ans.* £112 10s. New-England.

6. Reduce £112 10s. New-England, &c. currency to New-York, &c.

$\frac{96}{72}=\frac{4}{3}$ *Ans.* £150 New-York, &c.

7. Reduce £100 10s. 6d. New-England, &c. currency to South Carolina and Georgia, &c.

$\frac{56}{72}=\frac{7}{9}$ £100 10s. 6d.
7

÷9=£78 3s. 8d. $2\frac{2}{3}$ Geo. &c. *Ans.*

8. Reduce £78 3s. 8d. $2\frac{2}{3}$qr. South Carolina and Georgia currencies to New-England, &c.

$\frac{72}{56}=\frac{9}{7}$ £78 3s. 8d. $2\frac{2}{3}$
9

÷7=£100 10s. 6d. New-Eng. &c.—*Ans.*

9. Reduce 150*l.* 10s. 4d. New-England, &c. currency to New-Jersey, &c.

$\frac{90}{72}=\frac{5}{4}$ 150*l.* 10s. 4d.
5

÷4=188*l.* 2s. 11d. New-Jersey, &c.—*Ans.*

10. Reduce 188*l.* 2*s.* 11*d.* New-Jersey, &c. currency to New-England, &c.

$\frac{72}{90}=\frac{4}{5}$ 188*l.* 2*s.* 11*d.*
4

———

÷5=150*l.* 10*s.* 4*d.* New-England, &c. *Ans.*

11. Reduce 120*l.* 6*s.* 8*d.* New-York, &c. currency to New-Jersey, &c.

$\frac{90}{96}=\frac{15}{16}$ 120*l.* 6*s.* 8*d.*
15

———

÷16=112*l.* 16*s.* 3*d.* New-Jersey, &c. *Ans.*

12. Reduce 112*l.* 16*s.* 3*d.* New-Jersey, &c. currency to New-York, &c.

$\frac{96}{90}=\frac{16}{15}$ 112*l.* 16*s.* 3*d.*
16

———

÷15=120*l.* 6*s.* 8*d.* New-York, &c. *Ans.*

13. Reduce 160*l.* 10*s.* New-York, &c. currency to Canada and Nova Scotia, &c.

$\frac{60}{96}=\frac{5}{8}$ 160*l.* 10*s.*
5

———

÷8=100*l.* 6*s.* 3*d.* Canada, &c. *Ans.*

14. Reduce 100*l.* 6*s.* 3*d.* Canada and Nova Scotia, &c. currency to New-York, &c.

$\frac{96}{60}=\frac{8}{5}$ 100*l.* 6*s.* 3*d.*
8

———

÷5=160*l.* 10*s.* New-York, &c. *Ans.*

15. Reduce 150*l.* 15*s.* 6*d.* South Carolina and Georgia currency to New-York, &c.

$\frac{96}{56}=\frac{12}{7}$ 150*l.* 15*s.* 6*d.*
12

÷7=258*l.* 9*s.* $5\frac{1}{7}$*d.* New-York, &c. *Ans.*

16. Reduce 258*l.* 9*s.* $5\frac{1}{7}$*d.* New-York, &c. currency to South Carolina and Georgia, &c.

$\frac{56}{96}=\frac{7}{12}$ 258*l.* 9*s.* $5\frac{1}{7}$*d.*
7

÷12=150*l.* 15*s.* 6*d.* South Carolina and Georgia, &c. *Ans.*

Questions relative to the Reduction of Currencies.

1. At how much was the value of a Spanish milled dollar estimated, in the early settlement of the American Colonies?

2. What caused the different valuations of a dollar, or of pounds, shillings, and pence, in the different States, soon after their settlement, down to the present time?

3. How many shillings, or shillings and pence, now constitute a dollar in the several States in the Union? also in Canada, Nova Scotia, and sterling?

4. How are the currencies of the different States reduced to *Federal Money,* where the dollar is an even number of shillings, and the sum consists of pounds only?

5. What is the reason for the foregoing rule?

6. What is the rule, when the given sum consists of pounds, shillings, and pence?

7. What is the reason for this rule?

8. What is the rule for reducing New-Jersey, &c. currency to Federal Money?

9. On what principles is this rule constructed?

10. What is the rule for reducing South Carolina and Georgia currency to Federal Money? and why is such a rule established?

11. What is the rule for reducing Canada and Nova Scotia currency to Federal Money? and why this rule?

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bear to each other. This involves the principles of Arithmetical and Geometrical proportions.

It hence becomes necessary, in the regular advancement of the mind in the science of numbers, to attend to the general principles of proportion, previous to entering upon the Rule of Three. Without some knowledge of this, the learner must labour under great embarrassments, in acquiring clear and discriminating ideas of the subsequent rules, which are founded almost entirely on the nature of proportion.

OF PROPORTION IN GENERAL.

Numbers are compared together, in order to discover the relation which they bear to each other.

To form a comparison, there must necessarily be two numbers; the number which is compared, being written first, is called the *antecedent;* and that with which it is compared, the *consequent.*

Numbers are compared with each other in two different ways. The one comparison considers the *difference* of the two numbers, and is called arithmetical relation, and the difference is frequently termed the arithmetical ratio: the other comparison considers their *quotients,* viz. the geometrical ratio.

Thus, of the numbers 4 and 12, the difference, or Arithmetical ratio is 8; as 12—4=8; but the geometrical ratio is 3, as 12÷4=3.

When two or more couplets or numbers have equal ratios, or differences, the equality is denominated proportion; and their terms are similarly placed; that is, either all the greater, or all the less are taken as *antecedents;* and the others as *consequents,* and they are thus called proportionals. The two couplets, 2, 4, and 6, 8, are taken thus;—2, 4, 6, 8; or thus,—4, 2, 8, 6, and are arithmetical proportionals. But the two couplets, 2, 4, and 8, 16, taken thus;—2, 4, 8, 16; or, 4, 2, 16, 8, are geometrical proportionals.

The arithmetical ratio increases or decreases, by a *common difference.* The geometrical ratio increases by a *common multiplier,* or decreases by a *common divisor.*

Ratios are ever considered as the result of the greater term of comparison diminished, that is, either subtracted or divided by the less term, without regarding either term as the antecedent.

In *arithmetical progression*, any rank of numbers more than two, increasing or decreasing by a common difference, viz. by a continual addition or subtraction of any equal or given number, is said to be an arithmetical progression: as 1, 2, 3, 4, 5, 6, is an ascending series; and 6, 5, 4, 3, 2, 1, is a descending series; or thus;

2, 4, 6, 8, 10, is an ascending arithmetical series;
10, 8, 6, 4, 2, is a descending arithmetical series.

In the first rank of numbers, viz. 1, 2, &c. the common difference is 1, whether by addition or subtraction: in the second rank of numbers, the common difference is 2, whether by addition or subtraction.

The numbers, which form the series, are called the *terms* of progression. The first and last terms are called the *extremes*. In any series of numbers in arithmetical progression, the *sum* of the two extremes is equal to the *sum* of any two means equally distant from them, as may be seen in the above series:—as 1+6=7; 2+5=7; and 4+3=7: or, 2+10=12; 4+8=12; and 6+6=12: the same is true both of the ascending and descending series.

When the number of terms is odd, the double of the middle term is equal to the sum of the two extremes, or to that of any two terms equally distant from the middle term, as above, "6+6=12."

Respecting *geometrical progression*, when any rank or series of numbers is increased by one common multiplier, or decreased by one common divisor, such series belong to geometrical progression. For example; 1, 2, 4, 8, 16, 32, constitute a series which is increased by the common multiplier 2. Again, 81, 27, 9, 3, 1, is a series decreased by the common divisor 3.

When the terms of any series of numbers are in geometrical progression, the *product* of the two extremes will be equal to that of any two means equally distant from the extremes; and if the terms be odd, the middle term multiplied into itself will

be equal to the product of the two extremes, or that of two means equally distant from the middle terms: as 2, 4, 8, 16, 32; here, $32\times2=64$; $16\times4=64$; and $8\times8=64$. Also, 1, 3, 9, 27, 81; now $81\times1=81$; $27\times3=81$; and $9\times9=81$.

This brief explanation of the nature of proportion, both arithmetical and geometrical, with the obvious distinctions by which they are respectively characterized, will essentially aid the learner in his progress in the subsequent rules. This is clearly evident from the consideration, that the Rule of Three; or as it is often denominated, the Rule of Proportion, has its radical foundation on the nature and principles of geometrical progression.

Questions relative to Proportion.

1. To discover what, are numbers compared together?
2. How many numbers are required to form a comparison?
3. By what name, is the number compared and first written, called?
4. By what name is that called, with which it is compared?
5. In how many ways are numbers compared with each other?
6. What does the one comparison consider, and what is it called?
7. What is the difference often called?
8. What does the other comparison consider, and what is it called?
9. What is the difference, or arithmetical ratio of 4 and 12?
10. What is the geometrical ratio of 4 and 12?
11. When two or more couplets or numbers have equal ratios or differences, what is the equality denominated, and how are their terms placed?
12. Must all the greater or all the less terms be taken as *antecedents*, and the rest as *consequents?*
13. When thus taken, what are they called?
14. What proportionals are 2, 4, 6, 8? or 8, 6, 4, 2? or 4, 2, 8, 6?
15. What proportionals are 2, 4, 8, 16? or 16, 8, 4, 2? or 4, 2, 16, 8?

16. In what manner does an arithmetical ratio increase, or decrease?

17. How does a geometrical ratio increase, or decrease?

18. Of what are ratios ever considered as the result, without regarding either term as the antecedent?

19. What kind of progression may any rank of numbers more than two, increasing or decreasing by a common difference, be called?

20. Of what series, ascending or descending, are 1, 2, 3, 4, 5, 6? or, 6, 5, 4, 3, 2, 1? also, 2, 4, 6, 8, 10? or 10, 8, 6, 4, 2?

21. In the first rank of numbers, whether of ascending or descending series, what is the common difference, whether by addition or subtraction?

22. In the second rank, what is the difference, whether by addition or subtraction?

23. What are the numbers which form the series called?

24. What are the first and last term of the series called?

25. In any series of numbers in arithmetical progression, what is the sum of the two extremes equal to?

26. When the number of terms is odd, what is the double of the middle term equal to?

PROPORTION; OR

RULE OF THREE DIRECT.

IT is called the *Rule* of *Three*, in consequence of *three* numbers being given to find a fourth, which will be proportional; viz. which will bear the same proportion to the third, as the second does to the first; or, the fourth term will bear the same proportion to the second, which the third does to the first.

Of the three terms given, two are called terms of supposition, and the other that of demand. There are always two of the numbers given in a question, which are of the same name or

kind, one of which is the first, and the other the third term, in stating the question: the remaining number, which is of the same name or kind with the answer sought, will possess the second place.

In stating a question in the Rule of Three Direct,

1. Put that number in the third place, the value of which is sought, and which would follow some such inquiry, viz. what will? what cost? how many? how far? how long? how much? &c.

2. Place the term of the same name or kind with the third, in the first place; and if these terms be of different denominations, reduce them to the lowest denomination, in either of the given terms.

3. Put the remaining term in the second place, which will be of the same name with the answer; and if it consist of several denominations, reduce it to the lowest mentioned. Having thus stated the question, proceed by the following

Rule.

Multiply the second and third terms together, and divide that product by the first, and the quotient will be the fourth term, or answer; and of the same denomination with that of the second, or to which the second was reduced. When there is a remainder, multiply it by the next lower denomination, and divide by the first term, &c.

The proof is easily found by inverting the question.

Note.—There are various methods by which the work may be abridged, and which are oftentimes preferable to the general rule.

1. Divide the second term by the first, multiply the quotient into the third, and the product will be the answer. Or,

2. Divide the third term by the first, multiply the quotient into the second, and the product will be the answer. Or,

3. Divide the first term by the second, and the third by that quotient, and the last quotient will be the answer. Or,

4. Divide the first term by the third, and the second by that quotient, and the last quotient will be the answer.

So in the first example, under this rule, $8\times12=96$, is a product as much too great, as the first term 4, contains more units than 1. Therefore, $96\div4=24$, the fourth term proportional. Then $4\times24=96$; and $8\times12=96$; so that the result is, 4 : 8 : : 12 : 24.

The same will be true of every operation in the Rule of Three, whether direct or inverse; the product of the means and extremes will be equal. For, whether the divisor be the first, or third term, as the statement shall fall under direct or inverse proportion, the effect will be the same, and the true answer obtained.

These several examples may also be wrought by the rules given in the last note. Take the first question, and obtain the answer agreeably to the first rule, viz.; divide 8 by 4, and multiply the quotient 2 by 12, the answer is 24; by the second rule, $12\div4\times8=24$, answer: or take the second question; and by rule third, divide the first term 24, by 12, the second term, and divide the third term 8, by the last quotient: thus, $24\div12=2\div8=4$, answer. Or the same example by the fourth rule. Divide the first by the third term, and the second term by this quotient, and the last quotient is the answer. Thus, $24\div8=3\div12=4$, answer.

Example.

5. If 5*cwt.* of sugar cost £18 15*s.* what will 56*lb.* cost?

Reduced 5*cwt.* : £18 15*s.* : : 56*lb.*

```
 lb.     s.        lb.     s.   d.
 560  :  375  :  :  56  :  37 : 6
          56
        ----
        2250
        1875
        ----
56|0)2100|0(37
      168
      ---
       420
       392
       ---
        28
        12
       ---
   56)336(6
      336
      ---
```

Here the first and third terms are *lbs.* and the second term reduced to its lowest denomination, shillings.

Ans. £1 17*s.* 6*d.*

6. If 3 pairs of stockings cost 13*s.* what will 6 dozen pairs cost?

prs. *s.* 6 doz. is pairs 72.
3 : 13 : : 72 : 312*s.*

```
        72
        13
       ---
       216
       72
       ---
     3)936
       ---
   2,0)31,2  shillings.
       ----
    £15,12 Ans.
```

7. At 1*s.* 6*d.* a pound, what will 138*lb.* of butter cost?

lb. *d.* *lb.* £ *s.* *d.*
1 : 17 : : 138 : 9 15 6 $138\times17=2346d.$

8. If a person spend 4*s.* 6*d.* a day, how much is that in a year?

d. *d.* *ds.* £ *s.* *d.*
1 : 54 : : 365 : 82 2 6 $54\times365=19710$

9. If 7 days' board cost 22*s.* what will it come to by the year?

d. *s.* *d.* £ *s.* *d.*
7 : 22 : : 365 : 57 7 $1\frac{5}{7}$ $365\times22\div7=1147s.$ $1\frac{5}{7}d.$

10. If a man's salary be £250 a year, what is that by the calendar month?

M. £ *M.* £ *s.* *d.*
12 : 250 : : 1 : 20 16 8

11. How much will a grindstone, 30 inches in diameter, and 5 inches thick, come to, at 5*s.* per cubic foot?

Diameter, $30+15\times15\times5=3375$ Cubic inches.

Then, 1728 : 5*s.* : : $3375\times5\div1728$: $=9s.$ $9d.+$ *Ans.*

12. What is the expense of a grindstone, 34 inches in diameter, and 4 inches thick, at 6*s.* the cubic foot?

Diameter, $34+17\times17\times4=3468$

Then, 1728 : 6*s.* : : 3468 : $=12s.$ $0d.$ $2qr.$ *Ans.*

13. What will 52 thousand 4 hundred and 25 casts of staves cost, at $16 per thousand?

Note.—Staves are counted by casting 3 at a time: 40 casts make 1 hundred, and 10 hundred 1 thousand. Marked thus, *m.* thousand; *h.* hundred; *c.* casts.

	m.		*$*		*m. h. c.*
As	1	:	16	: :	52 4 25
	10				10
	10				524
	40				40
	400				20985

20985×16÷400=$839 40*cts.* *Ans.*

14. What will 25*m.* 9*h.* 35*c.* of staves cost, at $17 per thousand? *Ans.* $441,065+

15. What will 38*m.* 6*h.* 35*c.* of staves amount to, at $14 per thousand? *Ans.* $541,625.

16. What is the cost of 56*m.* 5*h.* 12*c.* of staves, at $14 15*cts.* per thousand? *Ans.* 819,685.

17. What is the cost of 25*m.* 750*ft.* of boards, at $16 50*cts.* per thousand?

$ *cts.* $ *cts.*
1000 : 16,50 : : 25750 : *Ans.* 424,875.

18. When boards sell at $20 per thousand, what is it per foot? *Ans.* ,02*cts.*

Note.—Hoops are frequently sold by bundles of 30 hoops each; and 4 such bundles make 1 hundred, and 10 hundred, or 40 bundles, make 1 thousand. But in some instances they are bound in bundles of 40 each, 3 bundles making 1 hundred, and 10 hundred, or 30 bundles, 1 thousand.

19. What is the cost of 67 bundles of hoops, at $24 per thousand of 30 bundles?

bun. $ *bun.*
As 30 : 24 : : 67 : 67×24÷30=$53,60 *Ans.*

20. What is the cost of 87 bundles, at $26 per thousand of 40 bundles?

bun.	$	*bun.*	
40 :	26 : :	87	or, 87×30×26÷1200=$56,55 *Ans.*

Note.—The number of hoops in either case is equal, viz. 1200 make 1 thousand. It is immaterial whether 30 is multiplied by 4, or 40×3; each will produce 120; and 120×10=1200, or 1 thousand.

Note.—In *Federal Money*, proceed as in whole numbers, only be careful to preserve the separatrix in its proper place; or agreeably to the rules of decimals.

21. If 3 hogsheads of sugar, each weighing 8*cwt.* 2*qr.* 14*lb.* cost $289 80*ct.* how much is that by the 7*lb.*

Three hogsheads are equal to 2898*lb.*

	lb.	$	*lb.*	*ct.*
Then, as	2898 :	289,80 : :	7 :	,70 *Ans.*

22. If 9 yards of cloth cost $11,25, how much cloth can be bought with $121,25?

ct.	*yd.*	*ct.*	*yd.*
1125 :	9 : :	12125 :	97 *Ans.*

23. If 3*lb.* of sugar cost 31½*ct.* what will 109*lb.* cost?

lb.	*ct.m.*	*lb.*	$ *ct. m.*
3 :	,315 : :	109 :	11,44,5.

24. If 15*lb.* of cheese, cost $1,42½*ct.* what will be the cost of 1*cwt.* at the same rate or ratio?

lb.	$ *ct.m.*	*lb.*	$ *ct.*
15 :	1,42,5 : :	112 :	10,64

Note.—The method to which we have briefly attended, in the Rule of Three Direct, is the same which has been generally practised, until within a few years past. It would doubtless be the most feasible to the learner, were it not, that where *more*

requires *less*, or *less* requires *more*, such questions would belong to the Rule of Three Inverse, which requires a different operation, viz.; after stating the question as before, to multiply the first and second terms together, and divide that product by the third. The object of the rule, which has more recently been adopted by a few arithmeticians, is, to do away the distinctions of direct and inverse proportion, by an alteration of the positions of the given terms. They would have the first and second terms of the same name or kind; and also the third and fourth to be similar. The effect is precisely the same; and one uniform rule of multiplying and dividing the terms, gives the true result. Under these circumstances, I deem it proper to give both methods, and leave it optional with the learner which to adopt, when he shall have become acquainted with both. The latter is evidently the most expeditious; and I doubt not, but it will shortly gain an entire ascendency over the former. The old rule of inverse proportion, I shall therefore defer, until some attention has been devoted to the modern method.

The following rule, to which reference is had, is thus defined:—

THE

SINGLE RULE OF THREE.

THIS teaches, by having three numbers given, to find a fourth, which will be in proportion to the third, as the second is to the first. Consequently there are *two* terms of *supposition*, and *one* of *demand*.

Rule for stating.

1. Put that term in the third place, which is of the same name or kind with the answer sought; that is, if the answer be money, the third term must be money; if the answer be measure, weight, or distance, the third term must be measure, weight, or distance, according as the demand shall require.

2. Then reflect, from the nature of the question, whether the answer must be greater or less than the third term. If greater, then put the larger of the two remaining numbers in the second place, and the less in the first; but if the answer required, be less than the third term; then put the less number of the two in the second place, and the greater in the first.

Rule for working.

3. If the first and second terms be of different denominations, reduce both terms to the lowest denomination in either of the terms.

4. If the third be composed of several denominations, reduce it to the lowest one named.

5. Then multiply the second and third terms together, and divide the product by the first, and the quotient will be the fourth term, or the answer sought; and it will also be of the same denomination as that to which the third term was reduced.

Should there be a remainder, multiply it by the next lower denomination, and divide the product by the first term.

The *proof* is obtained by inverting the question, and making the answer the third term.

Examples.

1. If 6 bushels of wheat cost $5 25*ct.* what will 70 bushels cost?

Stated thus:

```
B.   b.        $
6 : 70  : :  5,25
               70
           -------
          6)367,50
           -------
         $ 61,25 Ans.
```

2. If 100*lb.* of butter cost $12,50, what are 18*lb.* worth?

```
lb.   lb.      $
100 : 18  : :  12,50
                  18
               -----
               10000
               1250
               -----
        1|00)2,25|00
Ans. $2,25.
```

3. If a person's salary be $875 for 52 weeks, how much is it per week?

```
W.   w.       $         $ ct. m.
52 : 1  : :  875   Ans. 16,82,6+
```

Note.—In the first example, one term is money, and two are bushels. Money is the answer sought, therefore the term money is put in the third place. By examining, it is seen the fourth term must be greater than the third; and hence the larger number of bushels is put in the second place.

In the second example, it is evident the answer must be less than the third term; hence the larger number is put in the first place.

4. If 47 yards of cloth cost $146,87½, what will 7 yards cost?

yd.	*yd.*		*$ ct. m.*		*$ ct. m.*
47 :	7	: :	146,87,5 :		*Ans.* 21,87,5

5. If a flock of 725 sheep cost $1268, 75, what will 125 cost?

s.	*s.*		*$ ct.*	*$ ct.*
725 :	125	: :	1268,75	*Ans.* 218,75.

6. How many yards of carpeting, 2⅓ feet wide, will cover a floor 20 feet long, and 18 feet wide?

Here multiply the 2*ft.* 4*in.* by 3, because 3 feet make a yard. Then there are two widths, and one length given. Hence the length takes the third place, thus:

ft. in.	*w.*	*w.*		*long.*		*yd.*
$2,4 \times 3 = 7$	.	18	: :	20	Then $18 \times 20 \div 7$	$51\frac{3}{7}$ *Ans.*

7. If a garrison of 550 men, are supplied with provisions for 8 months, how many men would the same quantity supply 5 months?

mo.	*mo.*		*men.*	*men.*
5 :	8	: :	550	*Ans.* 880.

8. If *A* loan *B* $550, for 6 months, how long must *B* loan to *A* $150, to repay his kindness?

$	*$*		*mo.*	*mo.*
150 :	550	: :	6	*Ans.* 22.

9. If a cistern have 4 pipes, each of which will empty it in an hour, how many pipes of equal capacity will empty it in 12 minutes?

m.	*h.*		*pi.*	*pi.*
12 :	4	: :	1 :	*Ans.* 20.

10. A cistern, capable of containing 1500 gallons of water, is hourly supplied by a pipe, with 120 gallons; but it has two leakages, one of which wastes 20, the other 40 gallons hourly: how long will it take to fill the cistern, all three running at the same time?

	gal.		*h.*		*h.*
20+40—120 :	1500=	:	1 :		*Ans.* 25.

11. What number of men will be required to do in 10 days, what 30 men would do in 25 days?

d.		*d.*		*m.*	*m.*
10	:	25	: :	30	*Ans.* 75.

12. If a meadow will supply hay for 20 horses, 12 weeks, how long would it supply 8 horses? *Ans.* 30*wk.*

13. If *B* is indebted to *C* \$2570, and *B* has only \$1606,25, which he delivers over to *C*, how much does it pay on the dollar? *Ans.* 62½*ct.*

14. What will 3 dozen of combs come to, if 46½ dozen cost \$159,03? *Ans.* \$10,26.

15. If \$4,75 is paid for a cord of wood, what is the cost of 37 cords? *Ans.* \$175,75.

16. If 18*oz.* 5*pwt.* of silver be valued at \$12,16; what is the value of 96*oz.* 12*pwt.*? *Ans.* \$64,35 6+

17. If a regiment of 648 men, is to be clothed; each suit containing 4½ yards, at £1 2*s.* 6*d.* per yard; how many yards would it require; and how much is the cost of the cloth?

Ans. 2916*yd.*; cost £3280 10*s.*

RULE OF THREE INVERSE.

That the learner may be made acquainted with the system which has long been adopted, respecting the Rule of Three, it becomes necessary to treat of it more particularly. An important distinction has been ade between direct and inverse proportion; and consequently the rules of operation were different. The criterion of judging whether the question belonged to direct or inverse proportion, was the following. "If more requires more, or less requires less, the question belongs to the

Rule of Three Direct." That is, if the second term is greater than the first, and requires the fourth to be greater than the third; then more requires more; and if the second term is less than the first, and requires the fourth to be less than the third; then less requires less, and in both cases the questions belong to direct proportion. *Example.* As 3 is to 6, so is 6 to 12. Here the second term 6 is double the first term 3, and 12 the fourth term is the double of 6 the third term. Again;—as 6 is to 3, so is 12 to 6. Here less requires less, and in similar proportions, and belongs also to direct proportion.

But if more requires less, or less requires more, the question belongs to the Rule of Three Inverse.

Examples.

If 2 men can accomplish a task in 4 days, how many days will it require 4 men to do it? *Ans.* 2 days.

In this question, more requires less, viz. the more men, the less time is required. Again:

If 4 require 2, how much time will 2 require? *Ans.* 4.

Here less requires more; viz. the less the number of men, the more days are required. This is therefore Inverse Proportion, and a different operation is required to obtain the answer.

RULE.

1. State and reduce the terms as in the Rule of Three Direct.

2. Multiply the first and second terms together, and divide the product by the third: the quotient will be the answer in the same denomination as the middle term was reduced to.

If there is any remainder, multiply it by the next lower denomination, and divide the product by the third term.

Proof as in Direct Proportion.

Examples.

1. If 1 rod wide, and 160 long, make an acre; what length will be required, when the width is 5 rods, to make an acre?

$$\begin{array}{lccccl} & w. & l. & & w. & \\ \text{As } & 1 : & 160 & :: & 5 : & \quad \textit{Ans. } 32 \text{ rods.} \end{array}$$

2. If 90 bushels of corn, at 60 cents the bushel, will pay a given sum; how many bushels of wheat, at $1 per bushel, would pay the same?

ct. *bu.* *ct.*
As 60 · 90 : : 100 :
60

1|00)54|00(*Ans.* 54 Bushels of wheat.

3. If a man performs a journey in 12 days, when the days are 14 hours long; how many days will he be in performing the same, when the days are 8 hours long?

h. *d.* *h.* *d.*
As 14 : 12 : : 8 *Ans.* 21.

4. If the above traveller should perform his tour in 12 days, the days being 14 hours long; how many hours long must the day be to accomplish the same in 21 days?

d. *h.* *d.* *h.*
As 12 : 14 : : 21 : *Ans.* 8.

5. If 70 men, in 30 days, can dig a trench; how many men will it require to do the same work in 12 days?

d. *m.* *d.* *m.*
As 30 : 70 : : 12 : *Ans.* 175.

6. If it require 75 men, 15 days, to build a fortress; how many days would it require 375 men, to do the same?

m. *d.* *m.* *d.*
As 75 : 15 : : 375 : *Ans.* 3.

7. If $20 will pay the freight of 15*cwt.* 175 miles; how far may 25*cwt.* be carried for the same sum?

cwt. *m.* *cwt.* *m.*
As 15 : 175 : : 25 : *Ans.* 105.

8. If a board be 8 inches wide; how long must it be to measure 20 square feet?

in. *sq.ft.* *in.* *ft.*
As 12 : 20 : : 8 : *Ans.* 30.

9. How much in length, that is 6 inches wide, will make a square foot? *Ans.* 24*in.*

10. If 320*lb.* be suspended at the distance of 3 quarters of an inch from the centre of motion in a steelyard; what will be the weight of a poise, placed at the distance of 40 inches, to balance it?

	qr.		*lb.*		*in. qr.*		*lb.*
As	3	:	320	: :	40×4 :		*Ans.* 6.

11. If a floor be 20 feet wide, and 30 feet long; how much carpeting, 2 feet wide, will cover it? *Ans.* 100*yd.*

12. If 30 men can erect a building in 12 days; how many men would accomplish the same work in 120 days?—*Ans.* 3*m.*

The learner ought now to be capable of deciding, which method of stating questions in the Rule of Three, and of obtaining the answers, is the most easy and expeditious. By adopting the general rule, he is not encumbered with the distinctions of Direct and Inverse Proportions; nor is his mind burdened to retain the different rules applicable to each.

The Single Rule of Three, in Vulgar and Decimal Fractions, will next claim attention. The rules will be given in their various methods of operations; and the learner left to exercise his own choice in the selection of the rules. The modern rule will first be given; the others will follow in order.

THE

SINGLE RULE OF THREE

IN VULGAR FRACTIONS.

Rule.

Prepare the Fractions as directed for the Multiplication or Division of Vulgar Fractions; then state the question as in the

Rule of Three, in whole numbers. Invert the first term, and multiply all the three terms continually together, and the product will be the answer in the same name and kind as the third term. Proof as before.

RULE of THREE DIRECT, in VULGAR FRACTIONS.

Rule.

Prepare the Fractions as above: state the question as in whole numbers in the Rule of Three Direct. Invert the first term: then multiply all the three terms continually together, and the product will be the answer in the same name and kind with the middle term.

RULE of THREE INVERSE, in VULGAR FRACTIONS.

Rule.

Prepare the Fractions, and state the question as before: invert the third term, and multiply all the three terms together: the product will be the answer, corresponding with the middle term.

SINGLE *RULE of THREE, in DECIMAL FRACTIONS.*

Having reduced Fractions to their decimal expressions; the rules of operation are entirely correspondent with those of whole numbers, or Vulgar Fractions, whether in general rule, or those of direct, or inverse proportions. The separatrix must be carefully regarded.

Examples.

1. If $\frac{1}{2}$ a yard cost $\frac{5}{9}$ of a pound; what will $\frac{7}{8}$ of a yard cost?

$$\begin{array}{l} \quad\ \ \textit{yd.}\quad \pounds \qquad \textit{yd.} \\ \text{As } \tfrac{1}{2} : \tfrac{5}{9} :: \tfrac{7}{8} \end{array} \quad \text{And } \tfrac{2}{1}\times\tfrac{5}{9}\times\tfrac{7}{8}=\tfrac{70}{72}= \begin{array}{cccc} \pounds & s. & d. & qr. \\ 0 & 19 & 5 & 1\tfrac{1}{3} \end{array} \textit{ Ans.}$$

The above question stated by the general rule.

$$\begin{array}{l} \textit{yd.}\ \ \textit{yd.}\qquad \pounds \\ \tfrac{1}{2} : \tfrac{7}{8} :: \tfrac{5}{9} \end{array} \quad \text{And } \tfrac{2}{1}\times\tfrac{7}{8}\times\tfrac{5}{9}=\tfrac{70}{72}= \begin{array}{cccc} \pounds & s. & d. & qr. \\ 0 & 19 & 5 & 1\tfrac{1}{3} \end{array} \textit{ Ans. as before.}$$

2. If $\frac{1}{2}$ of a yard, cost $\frac{3}{7}$ of a pound; what will $\frac{9}{15}$ of an ell English cost?

Here are different denominations, of the same kind, which must be reduced, so as to correspond in denomination.

$\frac{1}{2}$ a yard $=\frac{5}{8}$ of $\frac{4}{1}$ of $\frac{1}{5}=\frac{20}{40}=\frac{1}{2}$ Ell English.

E. e. £ — £ s. d. qr.
As $\frac{1}{2} : \frac{9}{15} :: \frac{3}{7}$ And $\frac{2}{1}\times\frac{9}{15}\times\frac{3}{7}=\frac{54}{105}=$ 0 10 3 $1\frac{5}{7}$ *Ans.*

The second question wrought by Decimal Fractions.

E. e. — e. e. £
$\frac{1}{2} : \frac{9}{15} :: \frac{3}{7}$. And $\frac{1}{2}=,5$; $\frac{9}{15}=,6$; $\frac{3}{7}=,4285$ Reduced to

E. e. £
Decimals, is stated thus:—As ,5 : ,6 : : ,4285

£ s. d. qr.
Then ,4285×,6÷,5=,5142=10 3 1,63.

The first question wrought by Decimal Fractions.

Yd. yd. £
$\frac{1}{2} : \frac{7}{8}$ $\frac{5}{9}$ — $\frac{1}{2}=,5$; $\frac{7}{8}=,875$; $\frac{5}{9}=,5555$

Yd. yd. £ — s. d. qr.
As ,5 : ,875 : : ,5555 : Then, ,5555×,875÷,5=19 5 1,24+.

3. If $\frac{3}{4}$ of a yard, cost 67 hundredths of a dollar; what will $5\frac{3}{4}$ of a yard cost?

Yd. ct. yd. $
,75 : 67 : : 5,75 : *Ans.* 5,136+.

The same by Vulgar Fractions.

Yd. yd. ct. $
As $\frac{3}{4} : \frac{23}{4} :: \frac{67}{100}$ And $\frac{4}{3}\times\frac{23}{4}\times\frac{67}{100}=\frac{6164}{1200}=5,136+$ *Ans.*

4. If a person procured $6\frac{3}{4}$ yards of cloth, $1\frac{1}{4}$ yards wide, and he would line it with silk, $\frac{3}{4}$ of a yard wide; how many yards of silk must he purchase?

w. w. long.
As $\frac{3}{4} : \frac{5}{4} :: \frac{27}{4}$: And $\frac{4}{3}\times\frac{5}{4}\times\frac{27}{4}=\frac{540}{48}=\frac{45}{4}=11\frac{1}{4}$ yd. *Ans.*

The same by Decimals.

w. w. long.
As ,75 : 1,25 : : 6,75 : 6,75×1,25÷,75=11,25 yd. *Ans.*

5. If a cistern have three leakages, one of which will empty it in $\frac{1}{4}$ of an hour; 1 in $\frac{1}{2}$ an hour; and the third in $1\frac{1}{4}$ hours; in what time will it be emptied, if all three leakages are discharged at the same time?

This question will require two statements.

	h.		cis.		h.		cis.
1st. As	,25	:	1	: :	1	:	4
	,5	:	1	: :	1	:	2
	1,25	:	1	: :	1	:	,8
							6,8

2d. As 6,8 Cis. : 1 h. : 1 c.

Thus $1 \times 1 \div 6{,}8 =$,147 h.

,147
60
8|820
60
49 200

Ans. 8*m.* 49″.

6. If a conduit, discharging into a cistern, will fill it in 12 minutes; and the cistern has three discharging conductors, one of which will empty it in $\frac{1}{2}$ an hour; the second in 1 hour; and the third in $1\frac{1}{4}$ hour: in what time will the cistern be filled, if all four run together?

Two statements again. 1st.

	h.		c.		h.		c.	
As	,2	:	1	: :	1	:	5	The filling conduit gives 5 c. in an hour.
	,5	:	1	: :	1	:	2	The empt. conductors 3,8 c. in an hour.
	1,	:	1	: :	1	:	1	
	1,25	:	1	: :	1	:	3,8	1,2 difference.

Then $1 \times 1 \div 1{,}2 =$,8383 = 50*m.* 17″+ *Ans.*

7. If $\frac{7}{16}$ of a vessel, cost £96, what cost $\frac{3}{32}$ of it?

As $\frac{16}{7}$ v. : $\frac{96}{1}$ £ : : $\frac{3}{32}$: = *Ans.* £ 20 *s.* 11 *d.* $5\frac{1}{4}$.

Note.—If two of the terms, in the given question, be vulgar fractions, and the other a whole number; reduce the two fractional terms to a common denominator; then, rejecting the denominators, use the numerators, and the whole number in stating, as if all three terms were whole numbers; and the answer will be obtained.

8. If $\frac{5}{8}$ of a yard cost 12*s.*; what will $\frac{7}{12}$ of a yard cost?

$$\frac{5}{8}\ \frac{7}{12}\ \frac{60}{96}\ \frac{56}{96}\div 4=\frac{15}{24}\ \frac{14}{24}$$ Then, as *yd.* 15 : *s.* 12 : : *yd.* 14 : *Ans.* 11 *s.* 2 *d.* $1\frac{3}{5}$ *qr.*

9. If $\frac{7}{8}$ of a vessel is worth \$4500; what is $\frac{15}{16}$ of the vessel worth?

$$8\left|\begin{array}{cc} 7 & 15 \\ \hline 8 & 16 \\ \hline 1 & 2 \end{array}\right.\qquad \begin{array}{l} 16\div 8\times 7=14 \\ 16\div 16\times 15=15 \end{array}$$

	Ves.		\$		*w.*		\$	*ct.*	*m.*
Then, as	14	:	4500	: :	15	: *Ans.*	4821	42	8+.

The same by Decimals.

	Ves.		\$		*v.*		\$	*ct.*	*m.*
As	,875	:	4500	: :	,9375	: *Ans.*	4821	42	8+.

10. If a person own $\frac{5}{6}$ of a flour mill, and he sell $\frac{2}{3}$ of his share for £250; what is the value of the whole mill?

As *m.* 10 : *m.* 18 : : £250 : Then $250\times 18\div 10=$ £450 *Ans.*

11. If 6 days give 18*s.* $6\frac{3}{5}d.$; what will 30 days give?

As *d.* 6 : *s.* 18 *d.* $6\frac{3}{5}$: : 30*d.* *Ans.* £4 *s.* 12 *d.* 9

Note.—The middle term, after being reduced to pence, is then multiplied by 5, the denominator of the fraction, and 3 the numerator added to that product. After dividing the product by the first term, the quotient is then the fifth of a penny. This now is divided by 5, to bring it back to pence; then by 12 and 20, to get it into pounds. In all similar cases, the learner will bear in mind, the like operation must be observed, to save the fractional part.

12. If a board is $8\frac{5}{7}$ inches wide; how much in length must it be to make a square foot?

in. w. in. l.

As $\frac{144}{1} : \frac{1}{1} : : \frac{61}{7}$ And $\frac{144}{1}\times\frac{1}{1}\times\frac{7}{61}=\frac{1008}{61}=16\frac{32}{61}$ *ft.* *Ans.*

Promiscuous Examples in the Rule of Three

1. A sold a farm for £875 15*s*. of which he received $\frac{5}{8}$ of the money; what was his share? *Ans*. £547 6*s*. 10*d*. 2*qr*.

2. If $\frac{9}{16}$ of a vessel be worth \$328,375; what is the whole vessel worth? *Ans*. \$547 29*ct*. 1*m*.+

3. If a building 12 feet high, cast a shade on level ground 20 feet; how high is the spire of a steeple, the shade of which measures 199 feet? *Ans*. 119$\frac{2}{5}$ *ft*.

4. If the Legislature of a State grant a tax of 9 mills on the dollar; what must a person pay who is on the list \$416,82? *Ans*. \$2,75*ct*. 1*m*. 38+

5. How many pieces of Irish linen, each containing 20 ells Flemish, can be purchased for £23 8*s*.; at the rate of 6*s*. 6*d*. per ell English? *Ans*. 6 pieces.

6. If a merchant buy 90 ells French of silks, for £33 15*s*.; what did it cost per ell English? *Ans*. 6*s*. 3*d*.

7. If a hogshead of wine cost $\frac{1}{2}$ of $\frac{2}{3}$ of \$1000; what did the hogshead amount to; and what is the quantity of $\frac{1}{3}$ of $\frac{4}{5}$ of 30 gallons; and how much is it worth?

Ans. Cost of hhd. \$333 33*ct*. 3*m*. 8 gal. cost \$42 32*ct*. 8*m*.+

8. A merchant purchased 500 yards of cloth, for which he paid \$6 for every 10 yards, and sold it for \$11 for every 15 yards; did he gain or lose by the purchase?

Ans. He gained \$66 66*ct*. 6*m*.+

9. A merchant purchased a quantity of cloth to the amount of \$500; and in selling it out, he gained 3 cents on a yard, by which he made a net profit of \$150: how many yards did he purchase, and what did it cost him by the yard?

Ans. 5000*yd*. and gave 10*ct*. *yd*.

10. A merchant sold 6$\frac{1}{2}$ pieces of cloth, each piece containing 13$\frac{1}{4}$ yards, at 6*s*. 0$\frac{1}{4}$*d*. per yard: what was he amount?

Ans. £27 1*s*. 10$\frac{1}{32}$.

11. There are 800 men in a garrison, having a supply of provisions for 7 months: how many men must *leave* the garrison, that the same quantity of provisions may be sufficient for those who remain 10 months? *Ans*. 240 men.

12. A shipper purchased a quantity of wheat and oats for \$675. There were 400 bushels of wheat, at \$1,25 per bushel; and to every 4 bushels of wheat, there were 7 bushels of oats. How many bushels of oats were there, and what was the price of them by the bushel? *Ans.* 700*bus.* at 25*ct.* the bushel.

13. A pipe of wine cost \$189; but in removing it, 20 gallons leaked out: the residue was sold at \$1,87½ the gallon: did the purchaser lose or gain by the trade? *Ans.* He gained \$9,75.

Questions relative to the Rule of Three.

1. What is meant by the Rule of Three?
2. Upon what principle is this rule founded?
3. When are numbers said to be proportional?
4. What principles of proportion are involved in the Rule of Three?
5. Are the succeeding rules, in the system of Arithmetic, generally founded upon Proportion?
6. How has the Single Rule of Three been usually divided?
7. How are Direct and Inverse Proportions distinguished from each other?
8. When is more said to require more, and less to require less?
9. When is more said to require less, and less to require more?
10. In Proportion, what terms are a supposition, and what is the remainder called?
11. What terms in Direct Proportion are of the same name and kind, and must be reduced to the same denomination; and how are they disposed of?
12. Where is the remaining term placed; and what term must correspond with it both in kind and denomination?
13. After stating the question in Direct Proportion, how is the answer to be obtained?
14. Why is it that a true answer is thus obtained?
15. What has been the usual method of proof in the Rule of Three?
16. In Inverse Proportion, what terms have a relation to each other; and how is this relation?

17. Is the statement of a question, in Direct and Inverse Proportion, entirely similar, in every particular?

18. What is the difference, in working, to obtain the answer?

19. Is the why or wherefore, in obtaining the answer, in the least effected by the use of the first or third term for a divisor; as connected with the different statements?

20. To avoid the distinctions of Direct and Inverse Proportions, and to preserve entire uniformity in the multiplication and division of terms, what relative proportions, belonging to the Single Rule of Three, may be instituted?

21. Does this proportion admit of two terms of supposition, and one of demand?

22. What is the rule for stating questions in this proportion?

23. How is it known whether the greater or less, of the two similar terms, must occupy the first place?

24. What terms must be of the same denominations; and what of the remaining term?

25. Having stated and prepared the terms, how is the answer found? and of what kind and denomination?

26. Is the wherefore, relative to the true answer, the same as in Direct Proportion?

27. How is the common proof?

28. How are Vulgar Fractions prepared for the Rule of Three?

29. How is the statement made in the Rule of Three in Vulgar Fractions?

30. What is the rule for finding the answer?

31. Will all the various proportions be true of Vulgar Fractions, equally as of whole numbers?

32. How are the proofs?

33. How are Decimal Fractions stated and wrought in the Rule of Three?

34. Are the various proofs similar?

COMPOUND PROPORTION;

OR

DOUBLE RULE OF THREE.

This rule teaches to resolve, by one process, such questions as require two or more statings in single proportion. There are two methods of operation, viz.; ancient and modern. The former is the following:—there are (usually) five terms given to find a sixth, which, if the proportion be direct, will bear such proportion to the fourth and fifth, as the third bears to the first and second: but if inverse, the sixth will bear such a proportion to the fourth and fifth, as the first bears to the second and third. Of the five terms given, the three first are a supposition, and the two last a demand.

Rule.

In stating the question, place the terms of the supposition as follows: viz.—

1. Let that which is the principal cause of loss, gain, or action, possess the first place.

2. That which denotes time, distance, and the like, the second place.

3. The result of the cause, or remaining term in the third place.

4. Place the other two terms, or those of demand, under those of the same name or kind in the supposition.

5. If the blank fall under the third term, the proportion is direct; then multiply the first and second terms together for a divisor, and the other three for a dividend.

6. But if the blank fall under the first or second term, the proportion is inverse; then multiply the third and fourth terms together for a divisor, and the other three for a dividend.

Example.

If £100 in 12 months gain £6; what will £300 gain in 8 months?

The three first terms are a supposition; the two last a demand.

In the supposition, the £100 is the cause of gain, and takes the first place. Twelve, the next term, denoting time, is the second term. The effects, viz. 6 pounds, the third term. The terms 300 and 8 must stand under their respective denominations. Here the blank falls under the third term.

	£		m.		£
As	100	:	12	: :	6
	300	:	8		–

Then $300 \times 6 \times 8 = 14400$

And $100 \times 12 = 1200$

12|00)144|00

Ans. 12 months.

Proof by changing the statement of the question.

Note.—The reason of this rule may be easily seen, from that of the Single Rule of Three. It is observable in this case, that each line is a particular stating as in that rule; and therefore, if all the separate dividends be collected into one dividend, and all the divisors into one divisor, their quotients must be the answer sought, on similar principles.

The other, and perhaps most expeditious method of solving questions, under this rule, is the following:—

Rule.

1. Put that term, which is of the same name and kind with the answer, in the third place.

2. Then take one term from the supposition, and one from the demand, both being of the same name or kind, and place them in the line with the third term, as directed in the Single Rule of Three.

3. Proceed in the same manner with the two remaining terms.

4. Reduce the similar terms to the same denomination, if necessary.

5. Multiply the terms in the second and third places together, and divide their product by the product of those in the first place; the quotient or answer will be the term sought.

Proof by two statings; or invert the stating.

Note.—If either of the first terms, or both, will divide any of the three last, or can be divided by any of them, or by any other number, without a remainder, the operation may be contracted by cancelling them, and using their quotients in their stead.

Note.—Fractions, both Vulgar and Decimal, are subject to the same rules as whole numbers are.

Example.

1. If 4 men, in 8 days, earn $40; how much will 12 men earn in 24 days?

$$\left.\begin{array}{l}\text{Men } 4=12 \\ \text{Days } 8=24\end{array}\right\} \begin{array}{c}\$ \\ 40\end{array}$$

```
  32 288
      40
  ------
32)11520(360 dollars. Ans.
   96
   ---
   192
   192
   ---
     0
```

By contraction.

$$\left.\begin{array}{l}\text{Men } 4\div12=3 \\ \text{Days } 8\div24=3\end{array}\right\} \begin{array}{c}\$ \\ 40\end{array}$$

```
  9
 40
 ---
360 Ans.
```

Again, divide all by 4.

$$\left.\begin{array}{l}\text{Men } 4\div12=3 \\ \text{Days } 8\div24=6\end{array}\right\} \begin{array}{c}\$ \\ 40\end{array}$$

```
        2
   18
   40
  ----
 2)720
  ----
   360 Ans.
```

2. What principal, at 6 per cent, will give £20 in 8 months?

$$\begin{array}{lll} \text{Months} & 8 & 12 \\ \text{Pounds} & 6 & 20 \end{array}\Big\} 100 \quad \text{Then } 12\times20\times100\div8\times6=£500.$$

3. If \$100 dollars gain \$6 in a year; in what time will \$600 gain 6 dollars?

$$\begin{array}{ccc} \$ & \$ & m. \\ 600 & 100 & \\ 6 & 6 & \end{array}\Big\} 12 \qquad \textit{Ans. } 2 \text{ months.}$$

4. If 25 bushels of wheat are sufficient for 12 persons 8 months; how long will 50 bushels supply 18 persons?

$$\begin{array}{lcccccc} & bu. & & m. & & p. \\ \text{As} & 25 & : & 8 & : & 12 \\ & 50 & & — & & 18 \end{array} \quad \text{Then } 12\times50\div25\times8\times18=6m. \ \textit{Ans.}$$

5. If 40 men, in 20 days, accomplish a task; how many men will perform the same task, but 4 times as large, in 4 days?

$$\begin{array}{lll} \text{Days} & 4 & 20 \\ & 1 & 4 \end{array}\Big\} 40 \quad \text{Then } 20\times4\times40\div4=800m. \ \textit{Ans.}$$

6. If 10 men, in 12 days, mow 100 acres; how many men will mow 200 acres, in 20 days?

$$\begin{array}{ccc} m. & d. & a. \\ 10 & 12 & 100 \\ — & 20 & 200 \end{array} \qquad \textit{Ans. } 12 \text{ days.}$$

7. Suppose $8\frac{1}{2}$ men, in $6\frac{2}{3}$ days, earn $\frac{1}{2}$ of $\frac{1}{3}$ of £50; what will $15\frac{1}{4}$ men earn, in $10\frac{1}{2}$ days?

$$\begin{array}{ccccc} m. & d. & & m. & d. \\ 8\frac{1}{2} & 6\frac{2}{3} & \frac{1}{2} \text{ of } \frac{1}{3} \text{ of } £50 & 15\frac{1}{4} & 10\frac{1}{2} \end{array}$$

$$\begin{array}{ccccc} \frac{17}{2} & : & \frac{20}{3} & :: & \frac{50}{6} \\ \frac{61}{4} & & \frac{21}{2} & & — \end{array} \quad \text{Then } \frac{2}{17}\times\frac{3}{20}\times\frac{50}{6}\times\frac{61}{4}\times\frac{21}{2}= \overset{£}{23}\ \overset{s.}{10}\ \overset{d.}{11}\ \overset{qr.}{1\frac{15}{17}}.$$

The same by Decimal Fractions.

$$\begin{array}{ccccc} m. & & d. & & £ \\ 8,5 & : & 6,666 & :: & 8,333 \\ 15,25 & : & 10,5 & & — \end{array}$$

Then $8,333\times15,25\times10,5\div8,5\times6,666=£23\ 10s.\ 11d.\ 1qr.+$

Questions relative to the Double Rule of Three.

1. What is implied by the Double Rule of Three?
2. How many methods of operation are used to resolve questions by this rule?
3. How many terms are given, and what is to be found?
4. In direct proportion, what relation will the sixth term bear to the other terms?
5. In inverse proportion, what relation will the sixth term bear to the other terms?
6. Of the five given terms, which, and how many are a supposition; and which, and how many are a demand?
7. What is the rule for stating a question, which has long been practised?
8. If the blank fall under the third term, how is the proportion, and how is the operation to find the answer?
9. If the blank fall under the first and second terms, how is the proportion, and how is the operation to find the answer?
10. What is the reason of this rule?
11. What is the rule for stating, by the other method of solving questions?
12. When the question is stated, what is the rule of operation to find the answer?
13. How is the proof obtained?
14. Can the operation be shortened by division, or contraction, and how?
15. Are Fractions, both Vulgar and Decimal, subject to the same rules as whole numbers are?

PRACTICE.

Practice is a contraction of the Rule of Three, when the first term happens to be a unit or one. It derives its *name* from

its daily use among merchants, and others in business; and is a concise method of resolving most questions which occur in trade, where money is computed in pounds, shillings, and pence: but computation in Federal Money will render this rule almost useless. It will be needless, therefore, to be very minute on this subject.

Tables of Aliquot or Even Parts.

Parts of a Shilling.

d.		s.
6	is	$\frac{1}{2}$
4	- - - -	$\frac{1}{3}$
3	- - - -	$\frac{1}{4}$
2	- - - -	$\frac{1}{6}$
$1\frac{1}{2}$	- - - -	$\frac{1}{8}$

Parts of a Pound.

s. d.		£.
10	is	$\frac{1}{2}$
6 8	- - - -	$\frac{1}{3}$
5	- - - -	$\frac{1}{4}$
4	- - - -	$\frac{1}{5}$
3 4	- - - -	$\frac{1}{6}$
2 6	- - - -	$\frac{1}{8}$
1 8	- - - -	$\frac{1}{12}$

Parts of Two Shillings.

s.		s.
1	is	$\frac{1}{2}$
8*d.*	- - - -	$\frac{1}{3}$
6*d.*	- - - -	$\frac{1}{4}$
4*d.*	- - - -	$\frac{1}{6}$
3*d.*	- - - -	$\frac{1}{8}$
2*d.*	- - -	$\frac{1}{12}$

Parts of a Cwt.

lb.		cwt.
56	is	$\frac{1}{2}$
28	- - -	$\frac{1}{4}$
16	- - -	$\frac{1}{7}$
14	- - -	$\frac{1}{8}$
7	- - -	$\frac{1}{16}$

The aliquot part of any number is such a part of it, which being taken a given number of times, will exactly make the number; or, will exactly measure or divide it without a remainder: as 3 is an aliquot part of 15, and 7 of 28.

Case I.

When the price of an integer, whether it be one yard, pound, gallon, &c. is an even part of one shilling: call the given quantity so many shillings, as there are yards, pounds, &c.; then divide that quantity by the even part, and the quotient will be the answer in shillings, &c.

Or find the value of the given quantity, at 2*s.* the yard, &c., and divide the said value by the even part of 2*s.* which is designated by the price, and the quotient will be the answer in shillings, &c., which reduce to pounds.

Note.—To find the value of any quantity, at 2*s.* per yard, it is required only to double the unit figure for shillings, and the other figures will be pounds.

Examples.

1. What will 3750½ yards of binding amount to, at 1½ by the yard?

Here the number of yards are called so many shillings, and the half yard of course is 6*d.*, which makes the value 10⅔ times more than it really is. Hence dividing it by the price, which is the aliquot part of a shilling, viz. ⅛*s.* the quotient will be the true value.

	s.	*d.*	
1½ | ⅛ |	3750	6	
	468	9 3	Then 468÷20=*Ans.* £23 8*s.* 9*d* 3*qr.*

2. What will 624*lb.* of rice come to, at 4*d.* per *lb.*?

Ans. £10 8*s.*

3. What will 566¼ yards of tape come to, at 1*d.* yard?

	s.	*d.*		£.	*s.*	*d.*	*qr.*
1 | $\frac{1}{12}$ |	566	3	*Ans.*	2	7	2	1.

4. What will 822½ yards of cotton come to, at 6*d.* per yard?

Ans. £20 11*s.* 2*d.*

5. What will 324 yards of ribbon come to, at 3*d.* per yard?

Ans. £4 1*s.*

6. What will 956½ yards of gingham come to, at 6*d.*per yard?

Ans. £23 18*s.* 3*d.*

Case II.

When the price is an even part of a pound; call the given quantity so many pounds; and if there are parts in the given quantity, call them parts of a pound, and divide it by that even part of a pound; and the quotient will be the answer in pounds, &c.

Note.—This case depends entirely on the same principle as the last case.

Examples.

1. What will $240\frac{1}{2}$ yards amount to, at 2*s*. 6*d*. per yard?

2*s*. 6*d*.		£. *s*.
8	2*s*. 6*d*. | $\frac{1}{8}$ |	240 10
£1 0 0		30 1 3

Ans. £30 1*s*. 3*d*.

2. What will $482\frac{3}{4}$ yards come to, at 6*s*. 8*d*. per yard?
Ans. £160 18*s*. 4*d*.

3. What will $597\frac{1}{2}$ yards come to, at 5*s*. per yard?
Ans. £119 10*s*.

4. What will $126\frac{1}{2}$ yards come to, at 4*s*. per yard?
Ans. £25 4*s*. 9*d*. $2\frac{2}{3}$*qr*

5. What will $372\frac{3}{4}$ yards come to, at 1*s*. 8*d*. per yard?
Ans. £36 1*s*.

6. What will $537\frac{1}{2}$ yards come to, at 3*s*. 4*d*. per yard?
£89 10*s*. 10*d*.

Case III.

When the given price is any number of shillings under 20.

1. When the shillings are an even number, multiply the quantity by half the number of shillings, and double the first figure of the product for shillings, and the rest of the product will be pounds.

2. If the shillings be odd, multiply the quantity by the whole number of shillings, and the product will be the answer in shillings, which reduce to pounds.

Examples.

yd. *s*.	*yd*. *s*.
226 at 12	334 at 9
6	9
£135 12 *Ans*.	2|0)300|6
	£150 6 *Ans*.

728 yards at 4*s*. per yard. *Ans*. £145 12*s*

4. 844 yards, at 14*s.* per yard. *Ans.* £590 16*s.*
5. 513 yards, at 8*s.* per yard. *Ans.* £205 4*s.*
6. 427 yards, at 7*s.* per yard. *Ans.* £149 9*s.*
7. 335 yards, at 11*s.* per yard. *Ans.* £184 5*s.*

Case IV.

When the given price is pence, or pence and farthings, and not the even part of a shilling: call the given quantity as many shillings, and if necessary, parts of shillings; which divide by the greatest aliquot part of a shilling, contained in the given price; and take parts of the quotient for the remainder of the price; and the sum of these several quotients will be the answer in shillings, &c. which reduce to pounds.

Examples.

1. What will 327*lb.* of raisins come to, at 9¾*d.* a pound.

d.					
		327			
6	½	163	3		
3	½	81	9		
¾	¼	20	5	1	
2|0)		26|5	8	1	
		13	5	8	1	*Ans.* £13 5*s.* 8*d.* 1*qr.*

2. 416 yards, at 1¾*d.* per yard. *Ans.* £3 0*s.* 8*d.*
3. 228 yards, at 2¾*d.* per yard. *Ans.* £2 12*s.* 3*d.*
4. 754 yards, at 7½*d.* per yard. £23 11*s.* 3*d.*
5. 626 yards, at 5¾*d.* per yard. *Ans.* £14 19*s.* 11*d.* 2*qr.*
6. 548 yards, at 6¾*d.* per yard. *Ans.* £15 8*s.* 3*d.*
7. 436 yards, at 8½*d.* per yard. *Ans.* £15 8*s.* 10*d.*
8. 376 yards, at 7¼*d.* per yard. *Ans.* £11 7*s.* 2*d.*

Case V.

When the price is shillings, pence, and farthings, and not the aliquot part of a pound; multiply the given quantity by shillings, and take parts of the pence and farthings, as in the foregoing cases, and add them together; the sum will be the answer in shillings.

[illegible]

3. 8*cwt.* 3*qr.* 16*lb.* at $8 88*ct.* per *cwt.* *Ans.* $78 31*ct.* 5*m.*
4. 7*cwt.* 1*qr.* 21*lb.* at 2*l.* 18*s.* per *cwt.*–*Ans.* 21*l.* 11*s.* 4*d.* 1*qr.*
5. 11*cwt.* 2*qr.* 14*lb.* at 1*l.* 12*s.* 6*d.* per *cwt.*–*Ans.* 18*l.* 17*s.* 9*d.* 3*qr.*
6. 6*cwt.* 0*qr.* 7*lb.* at $7 48*ct.* per *cwt.* *Ans* $45, 3475.
7. 0*cwt.* 1*qr.* 21*lb.* at $12 50*ct.* per *cwt.*–*Ans.* $5 46*ct.* 8*m.* 75.
8. 15*cwt.* 2*qr.* 14*lb.* at $10 75*ct.* per hundredweight. *Ans.* $167 96*ct.* 8*m.* 75.

Questions relative to Practice.

1. What is Practice?
2. Whence is the name derived, and why?
3. Is this rule founded on the principles of proportion?
4. May this rule be considered as a contraction of the Rule of Three, when the first term is a unit or one?
5. Is the rule of Practice equally required in Federal Money, as in pounds, shillings, and pence?
6. What is the meaning of aliquot parts?
7. What is the rule for the first case, when the price of an integer is an even part of a shilling?
8. How is the value, at two shillings the yard, most readily found?
9. What is the rule for the second case, when the price is an even part of a pound?
10. What is the rule for the third case, when the shillings are under 20, and an even number?
11. How is the rule under the same case, when the shillings are odd?
12. What is the rule for the fourth case, when the price is pence, or pence and farthings, and not the aliquot part of a shilling?
13. What is the rule for the fifth case, when the price is shillings, pence, and farthings, and not the aliquot part of a pound?
14. What is the rule for the sixth case, when the price and quantity given are of several denominations?

TARE AND TRETT.

Tare and Trett are practical rules for deducting certain allowances which are made by merchants, and others, in buying and selling goods, &c. by weight. The following particulars are to be noticed.

1. *Gross Weight* is the whole weight of any sort of goods, together with the box, cask, or bag, containing them.

2. *Tare* is an allowance made to the purchaser for the weight of the box, cask, or bag, &c. containing the goods bought; and it is either so much per box, &c.;—at so much per cwt.; or at so much in the whole gross weight.

3. *Trett* is an allowance of *4lb.* on every 104*lb.* for waste, dust, &c.

4. *Cloff* is an allowance of *2lb.* on every *3cwt.* for the turn of the scale.

5. *Suttle* is what remains after one or two allowances have been deducted, and yet a farther deduction is to be made.

6. *Net Weight* is what remains after all allowances are made.

Case I.

When the question is an Invoice:

Add the gross weights into one sum, and the tares into another; then subtract the total tare, from the whole gross; the remainder will be the net weight.

1. What is the net weight of 6 hogsheads of tobacco, marked with the gross weight, as follows :—

	No.	cwt.	qr.	lb.	Tare.	
	1	5	3	12	105	
	2	6	1	18	99	
	3	6	2	21	100	
	4	5	3	24	102	
	5	6	0	16	96	
	6	5	3	27	103	
Whole gross		37	0	6	605	Tare.
Whole tare		5	1	17		
Net weight		31	2	17	*Ans.*	

2. What is the net weight of 5 barrels of sugar, number and weight, as follows :—

	No.	cwt.	qr.	lb.	Tare.	
	1	3	3	15	40	
	2	4	0	20	38	
	3	3	1	25	39	
	4	4	1	16	37	
	5	3	2	22	36	
Whole gross		19	2	14	190	Tare.
Whole tare		1	2	22		
Net weight		17	3	20	*Ans.*	

Case II.

When the tare is at so much per box, cask, bag, &c. multiply the tare of 1 by the number of bags, bales, &c. and the product is the whole tare, which subtract from the gross weight, and the remainder will be the net weight.

Examples.

1. In 12 firkins of butter, each weighing 3*qr.* 14*lb.*; tare 12*lb.* on each firkin; what is the net weight? *Ans.* 9*cwt.* 0*qr.* 24*lb.*

2. In 100 barrels of figs, each weighing 2*qr.* 16*lb.*; tare 9*lb.* per barrel; what is the net weight? *Ans.* 6300*lb.*

3. In 5 hogsheads of sugar, each weighing 9*cwt.* 2*qr.* 21*lb.* gross; tare 82*lb.* per hogshead; what is the net weight?

	cwt.	qr.	lb.
	9	2	21
			5
Gross	48	1	21
Tare	3	2	18
Net wh.	44	3	3 *Ans.*

4. What is the net weight of 6 tierces of rice, each weighing 3*cwt.* 3*qr.* 24*lb.*; the tare 40*lb.* each tierce?

	cwt.	qr.	lb.
	3	3	24
			6
Gross	23	3	4
Tare	2	0	16
Net	21	2	16 *Ans.*

5. In 20 bags of pepper, each weighing 75*lb.* gross, tare per bag, 3½*lb.*; how many pounds net? *Ans.* 1430*lb.* net.

6. What is the net weight of 20 hogsheads of tobacco, each weighing 8*cwt.* 2*qr.* 12*lb.*; tare 100*lb.* per hogshead?

Ans. 154*cwt.* 1*qr.* 4*lb.*

Case III.

When the tare is at a given rate per hundredweight:

Divide the gross weight by the aliquot part of a hundredweight, for the tare, which subtract from the gross, and the remainder will be net weight.

Examples.

1. What is the net weight of 56*cwt.* 3*qr.* 24*lb.* gross; tare 14*lb.* per hundredweight?

		cwt.	qr.	lb.	oz.
	14 | ⅛	56	3	24	0
Tare		7	0	13	8
Net		49	3	10	8 *Ans.*

2. What is the net weight of 7 hogsheads of tobacco, each weighing 6*cwt.* 2*qr.* 12*lb.* gross; tare 16*lb.* per hundredweight?

Ans. 51*cwt.* 2*qr.* 16*lb.*

3. What is the net weight of 12 barrels of pot ash, each weighing 312*lb.* gross; tare 12*lb.* per cwt. *Ans.* 3343*lb.* 8*oz.*

4. In 124*cwt.* 3*qr.* gross, tare 21*lb.* per hundredweight; what is the net weight? *Ans.* 101*cwt.* 1*qr.* 12*lb.* 4*oz.*

5. In 20 *bls.* of sugar, each weighing gross, 2*cwt.* 2*qr.*18*lb.* tare 16*lb.* per hundredweight; what is the net weight?

Ans. 45*cwt.* 2*qr.* 12$\frac{4}{5}$*lb.*

6. In 6 hogsheads of sugar, each weighing 9*cwt.* 1*qr.* 14*lb.* gross, tare 14*lb* per cwt.; what is the net weight, and what will it amount to, at $8 75*ct.* per hundredweight?

Ans. 49*cwt.* 0*qr.* 24*lb.* 8*oz.*=$430, 625.

Case IV.

When Trett is allowed with Tare.

1. Find the tare, which subtract from the gross, and call the remainder suttle.

2. Divide the suttle weight by 26, and the quotient will be the Trett, which subtract from the suttle, and the remainder is net weight.

Note.—The reason of dividing by 26 to get the trett is, that 4 times 26 make 104; therefore 4*lb.* is $\frac{1}{26}$ of 104, and thus gives the proper quotient.

Examples.

1. In 3 hogsheads of sugar, each weighing 8*cwt.* 2*qr.* 16*lb.* gross; tare 14*lb.* per hundredweight; and trett 4*lb.* on every 104; how much net weight?

	cwt.	*qr.*	*lb.*	*oz.*	
	8	2	16		
			3		
Gross	25	3	20		
Tare	3	0	27		
Suttle	22	2	21		
Trett		3	13	11	
Net	21	3	7	5	*Ans.*

2. In 20 *bls.* of sugar, each weighing 2*cwt.* 2*qr.* 18*lb.*; tare 16 *lb.* per cwt.; trett 4*lb.* on every 104; how much net?

Ans. 43*cwt.* 3*qr.* 12*lb.* 3*oz.*

3. In the fourth example, under the last case, from the net weight deduct the trett, viz. 4*lb.* on every 104, and see what is left. *Ans.* 97*cwt.* 1*qr.* 23*lb.* 10*oz.*

4. In the sixth example, under the last case, deduct the trett, viz. 4*lb.* on 104, and get the net weight.

Ans. 47*cwt.* 1*qr.* 8*lb.* 2*oz.*

5. In the third example, under the last case, deduct trett, and get the net. *Ans.* 3214*lb.* 15*oz*

6. In the second example, under the last case, deduct trett, and find the answer. *Ans.* 49*cwt.* 2*qr.* 17*lb.* 9*oz.*

Case V.

When Tare, Trett, and Cloff are allowed.

Deduct the tare and trett as before: then divide the suttle by 168, (because 2*lb.* is $\frac{1}{168}$ part of 3 cwt.) and the quotient will be the cloff, which subtract from the suttle, and the remainder will be net weight.

Examples.

1. In 7 hogsheads of tobacco, each weighing 10*cwt.* 2*qr.* 14*lb.* tare 104*lb.* per hogshead; trett, 4*lb.* on every 104; and cloff, 2*lb.* on every 3 hundredweight; how much net weight?

	cwt.	qr.	lb.	oz.	
	10	2	14		
			7		
Gross	74	1	14		
Tare	6	2	0		
Suttle	67	3	14÷26		
Trett	2	2	11	4	
Suttle	65	1	2	12÷168	
Cloff		1	15	8	
Net.	64	3	15	4	*Ans.*

2. What is the net weight of 50*cwt.* 2*qr.* 22*lb.*; tare 78*lb.* and the trett and cloff as usual? *Ans.* 47*cwt.* 3*qr.* 4*lb.* 10*oz*

Questions relative to Tare and Trett.

1. What are Tare and Trett?
2. What particulars are to be noticed under this head?
3. What is Gross Weight?
4. What is Tare, and how is it estimated?
5. What is Trett, and why used?
6. What is Cloff, and why used?
7. What is Suttle?
8. What is Net Weight?
9. What is the rule for case first, when the question is an Invoice?
10. What is the rule for case second, when the Tare is at so much per box, cask, &c.?
11. What is the rule for case third, when the tare is at a given rate per hundredweight?
12. What is the rule for case fourth, when Trett is allowed with Tare?
13. Why divide the Suttle by 26, to find the Trett?
14. What is the rule for case fifth, when Tare, Trett, and Cloff are allowed?
15. Why divide the Suttle by 168, to find the Cloff?

INTEREST.

Interest is of two kinds; Simple and Compound.

SIMPLE INTEREST.

Simple Interest is the premium paid by the borrower to the lender, for the use of the money lent. It is generally established by law, at a certain rate per cent. per annum, which in most of the United States, is fixed at 6 per cent.; viz. £6 for the use of £100, or $6 for the use of $100, for one year; in some

of the States, it is rated at 7 per cent., that is, $7 for the use of $100, one year.

Principal is the sum lent.

Rate or *Interest* is the sum agreed upon.

Amount is the principal and interest added together.

Note.—The rules for Simple Interest serve also to calculate commission, brokerage, insurance, purchasing stocks, or any concern in which property is rated at any given per cent.

Note.—Per cent. or centum, is by the hundred: per annum by the year.

To find the Interest of any given sum for one year.

RULE.

Multiply the principal by the rate per cent., and divide the product by a 100, and the quotient will be the answer: or, multiply the principal by the rate per cent., and cut off two right-hand figures of the product; the figures on the left-hand of the separatrix will be pounds, and those on the right, parts of pounds, which multiply by 20, and cut off as before; and if there is still a remainder, multiply by 12, and so on to the lowest denomination. The figures on the left-hand of the separatrixes will be the answer sought.

Note.—The principle upon which this rule is founded, is that of proportion. To divide by 100, or what is the same in effect, cutting off the two right-hand figures of the product, presupposes, that if the given question were stated at full length, as in the Rule of Three, the first term would be 100. Thus, (the time, one year being here understood,) if £100 gain £6, what will £200 gain? Then $200 \times 6 \div 100 = £12$: or, as $200 \times 6 = 1200$, cutting off the two right-hand figures, leaves the quotient 12, the same as would result from dividing 1200 by 100. Indeed, were the question stated at full length, it would belong to the Double Rule of Three; thus:—

$$\begin{array}{lll} £. & m. & £. \\ 100 & 12 & 6 \\ 200 & 12 & \end{array} \Bigg\} \text{ Then, } 6 \times 200 \times 12 = 14400 \div 100 \times 12 = 12. \text{ An. } 12$$

But when it is said, if £100 gain £6, what will £75 gain; a year is understood, and it would be needless to bring it into the statement. It will be seen then, that the method of computing interest, is in reality a contraction of the Double Rule of Three, and is greatly expedited by simply multiplying by the rate per cent., and then cutting off the two right-hand figures.

Examples.

1. What is the interest of £73 13*s*. 6½*d*. for one year, at £6 per cent.?

```
 £   s.  d. qr.
 73  13  6   2
             6
-----------------
4|42  1  3   0
   20
----
8|41
   12
----
4|95
    4
----
3|80       £  s. d. qr.
---- Ans.  4  8  4  3,8.
```

2. What is the interest of £350 10*s*. for a year, at 6 per cent?

```
  £    s.
 350  10
       6
---------
21|03  0
    20
   ---
   |60
    12
  ----
  7|20
  ----
```

Ans. £21 0*s*. 7*d*.

3. What is the interest of £742 15*s*. 10¾*d*. for one year, at 6 per cent.?

```
  £    s.  d.  qr.
 742  15  10   3
               6
-------------------
44|56  15   4  2
    20
  ----
11|35
    12
  ----
  4|24
     4
   ---
   |98
   — Ans. £44 11s. 4d.
```

4. What is the interest of £625 16*s*. 3*d*. for one year, at 7 per cent?

```
  £    s.  d.
 625  16   3
           7
--------------
43|80  13  9
    20
  ----
16|13
    12
  ----
 1|65
    4
  ---
 2|60
  ---
```

Ans. £43 16*s*. 1*d*. 2*qr*.

5. What is the interest of £378 8s. 4d. for one year, at 7 per cent.?

£	s.	d.
378	8	4
		7
26|48	18	4
20		
9|78		
12		
9|40		
4		
1|64		

Ans. £26 9s. 9d. 1,64qr.

6. What is the interest of £545 7s. 5½d. for one year, at 5 per cent.?

£	s.	d.
545	7	5½
		5
27|26	17	3½
20		
5|37		
12		
4|47		
4		
1|90		

Ans. £27 5s. 4d. 1,90qr.

Federal Money.

Rule.—Multiply the principal by the rate per cent., and place the separatrix in the product, as in Multiplication of Federal Money. The figures on the left-hand of the separatrix will be the interest in cents, and the first figure on the right will be mills, and the others decimals of mills.

7. What is the interest of $272 50ct. for one year, at 6 per cent.?

$ ct. ct. $ ct.
272,50×6=1635,00. *Ans.* 16 35

8. Required the interest of $741 93ct. for one year, at 7 per cent.

$ ct. ct. $ ct. m.
741,93×7=5193,51 *Ans.* 51 93 5+

9. Required the interest of $648 37½ct. for one year, at 5 per cent.

$ ct. ct. $ ct. m. m.
648,37½×5=3241,87+ *Ans.* 32 41 8¾, or 75.

Note.—If the principal is written down in cents, and that multiplied by the rate per cent., and the product divided by 100, as before, it will give the interest for a year in cents, and decimals of a cent.

10. Required the interest of $95 25*ct* for one year, at 6 per cent.

	ct.	ct. m.	$ ct. m.
Principal	9525×6=571|50÷1|00=	571 5 or	5 71 5 *Ans.*

11. Required the interest of $357 62*ct.* for one year, at 7 per cent.

35762×7=2503,34 *Ans.* $25 03*ct.* 3*m.* 4.

12. Required the interest of $437 36*ct.* for one year, at 5 per cent.

43736×5=2186,80 *Ans.* $21 86*ct.* 8*m.*

When the given principal is pounds, shillings, and pence, New-England or Virginia currency, viz. 6*s.* and the interest is required in Federal Money.

RULE.

Reduce the given sum to shillings, and the product is the answer in cents; and the pence are mills nearly.

The reason is, that the interest of one pound, (at 6*s.* to the dollar,) at 6 per cent. per annum, is equal to $\frac{1}{5}$ of a dollar, viz. 20 cents; that is, 20 cents for 20 shillings, or 1 cent for every shilling. If the currency is 8*s.* to the dollar; reduce the given sum to shillings, and deduct $\frac{1}{4}$ from the shillings, and it leaves the interest in cents, at 6 per cent. As $2 50*ct.* make 20 shillings in this currency, and 6 per cent. on $2 50*ct.* per annum, is 15 cents, it requires $\frac{1}{4}$ deduction from the shillings to give the interest in cents.

13. Required the interest of £154 12*s.* 6*d.* for one year in Federal Money, at 6 per cent.

£ s. d.	ct. m.		$ ct. m.
154 12 6×20=	3092 6	*Ans.*	30 92 6.

14. **Required the interest of 32*l.* 18*s.* 9*d.* for one year in Federal Money, at 6 per cent.**

l.	*s.*	*d.*		*ct.*	*m.*		*$*	*ct.*	*m.*
32	18	9×20=		658	8	*Ans.*	6	58	8.

15. Required the interest of 430*l.* 10*s.* New-York currency, for one year in Federal Money, at 6 per cent.

430*l.* 10*s.*×20=8610÷4=2152,5 8610—2152,5=6457,5.
Ans. $64 57*ct.* 5*m.*

To find the Simple Interest of any sum of Money, for any number of years, and parts of a year.

General Rule.

1. Find the interest of the given sum for one year.
2. Multiply the interest of one year by the given number of years, and the product will be the answer for that time.
3. If there be parts of a year, as months and days; work for the months by the aliquot parts of a year; and for the days by the Rule of Three, or by allowing 30 days to the month, and taking aliquot parts of the same.

Note.—By allowing the month to be 30 days, the interest of any ordinary sum will be found sufficiently exact for common use; but if the sum be very large, it may be stated thus:—as 365 days is to the interest of one year, so is the given number of days to the interest required.

16. What is the interest of 125*l.* 14*s.* 6*d.* for 5½ years, at 6 per cent.?

£.	*s.*	*d.*
125	14	6
		6
7|54	7	0
20		
10|87		
12		
10|44		
4		
1|76		

£.	*s.*	*d.*	*qr.*
7	10	10	1
			5
37	14	3	1
3	15	5	1
41	9	8	2

Ans £41 9*s.* 8*d.* 2*qr.*

18 Required the interest of $362 45*ct.* for 3 years, 5 months and 10 days, at 7 per cent.

```
         $ ct.
        362,45
             7
  m.  ———————
 |4|⅓|2537,15
             3
      ———————
  m.  761145
 |1|¼| 845716
  d.   211439
 |10|⅓  70479
      ———————
     87,39,084
      ———————
```

Ans. $87 39*ct.* 0*m.* 84.

19. Required the interest of $776 87½*ct.* for 2 years and 9 months, at 5 per cent. *Ans.* $106 82*ct.* 0*m.*312

20. Required the interest of $534 34*ct.* for 4 years, 8 months, and 25 days, at 6 per cent. *Ans.* $151 84*ct.* 1*m.*61

21. Required the interest of 416*l.* 7*s.* 6*d.* for 5 years, at 7 per cent. *Ans.* 145*l.* 14*s.* 7*d.* 2*qr.*

22. Required the interest of 1827*l.* from May 1st, 1827 to January 17th, 1828, at 7 per cent.

Interest for one year is 127*l.* 17*s.* 9½*d.*

As 365*d.* : 127*l.* 17*s.* 9*d.* 2*qr.* : : 262*d.* : *Ans.* 91*l.* 16*s.*

When the time is Months.

Rule.—Multiply the principal by half the number of months, and cut off two figures from the right of the product, and the quotient will be the interest at 6 per cent.

23. Required the interest of $200, for 18 months, at 6 per cent.

$200×9*m.*=18,00 *Ans* $18.

24. Required the interest of $327 42*ct.* for 16 months, at per cent.

$327 42*ct.*×8=*ct.*2619,36 *Ans.* $26 19*ct.* 3*m.* 6.

Note.—If the given months are not an even number, then take half the principal, and multiply it by the whole number of months, and the answer will be the same.

25. Required the interest of $250, for 9*mo.* at 6 per cent.

	$		$
	250		250
	6		
m.		Half	125
6 ½	15,00		9
3 ¼	7,50		$11,25 *Ans.*
	3,75		
Proof.	$11,25 *Ans.*		

26. Required the interest of $340 40*ct.* for 15*mo.* at 6 per cent.

	$ *ct.*		$ *ct.*
	340,40		340,40
	6		
m		Half	170,20
3 ¼	2042,40		15
	510,60		
			85100
	25,53,00		17020
	Ans. $25 53*ct.*	Cents	2553,00
			Ans. $25 53*ct*

When the given time is months, to find the interest at *any given rate per cent.* for those months.

Rule 1.—Find the interest of the given principal, at the rate per cent. required, for one year.

2. Multiply this interest by the given number of months.

3. Divide that product by 12, the number of months in a year, and the quotient will be the interest sought.

27. Required the interest of $520 45*ct.* for 15*mo.* at 4 per cent.

$. *ct.* 1 *year.* $ *ct. m.*
520,45 × 4 = 20,81,80 × 15 ÷ 12 = 26 02 2 25 *Ans.*

28. Required the interest of \$350 60ct. for 7 months, at 7 per cent.

```
           $ ct.
          350,60
               7
          ------
Int. 1 yr. 24,54,20
               7 Months.
          ------
Mon. 12|17179,40
          ------
          1431,61
          ------
```

Ans. \$14 31*ct.* 6*m.* 1.

29. Required the interest of \$120, for 10 months, at 5 per cent.

```
          $
         120
           5
         ---
1 year  6,00
          10 Months.
         ---
     12)6000(5,00
        60
        --
```

Ans. \$5 00.

30. Required the interest of \$295 77*ct.* for 13*mo.* at 6 per cent.

\$ *ct.* 1 *year.* \$ *ct. m.*
295 77×6=17,74,62×13÷12=19 22 5 *Ans.*

31. Required the interest of \$487 32*ct.* for 17*mo.* at 3 per cent.

\$ *ct.* 1 *year.* \$ *ct. m.*
487,32×3=1461,96×17÷12=20 71 1 1.

When the time is *months and days*, and the annual interest is 6 *per cent.* to find the answer by one operation.

Rule.

Multiply the given principal by half the number of months, and one-sixth of the days; or, if more convenient, multiply half the given principal by the whole numbers of months, and one-third of the days, and the product will be the answer.

The figures expressive of days are decimals, and will be noticed as such in placing the separatrix. Should the days be 1 over the number thirded, add a 3; and if 2 days over, add a 6 to the right of the decimal for days. The figures on the left of the separatrix will be cents. If there are years in the given question, bring them into months.

Note.—The reason of this rule is obvious. If the given principal be multiplied by the whole number of months, the interest will be 12 per cent. Example.—100×12, and divided by 100,

gives 12 quotient. But 50×12=6; or 100×6=6, or 6 per cent. It is hence immaterial whether the principal or the time be halved to obtain 6 per cent. Thirty days are usually called a month, and are composed of 3 tens. One-third, therefore, brings the days into a proper ratio with months: so that, if months be halved, the third of the days must be halved.

Any required per cent. is easily obtained from 6 per cent. If 7 be required, divide the 6 per cent. by 6, and add this sixth part to the 6 per cent; or, if 8 per cent. divide the 6 per cent. by 3, there being 3 twos in 6, and add the quotient to the 6, and the answer is obtained at 8 per cent.

Examples.

31. Required the interest of $281 72*ct.* for 22 months and 18 days, at 6 per cent.

Half the prin.	140,86
Wh. No. of mo.	22,6
and 3d. do.	
	84516
	28172
	28172
Cents	3183,436

	$ *ct.*
	281,72
	11,3
	84516
	28172
	28172
Cents	3183,436

Ans. $31 83*ct.* 4*m.* 36.

32. Required the interest of $457 93*ct.* for 2 years, 4 months, and 24 days, at 6 per cent.

$ *ct.*	
457,93	
14,4	
183172	
183172	
45793	
6594,192	*Ans.* $65 94*ct.* 1*m.* 92.

33. Required the interest of 285*l*. 16*s*. 8*d*. for 3 years, 7 months, and 27 days, at 6 per cent.

```
Half the principal £142,9125 Decimal of a pound.
                        43,9 Whole number of months,
                     ———————      and 3d. of days.
                    12862125
                    4287375
                   5716500
                  ——————————
                  62|73,85875
                           20
                  ——————————
                   14|7717500
                           12
                  ——————————
                    9|2610000
                            4
                  ——————————
                    1|0440000
                  ——————————      Ans. £62 14s. 9d. 1qr.
```

Note.—The decimal for the days is easily found, by dividing decimally the given days by 3 or 6, as the case may require. It will be remembered, that the whole of the quotient will be decimals, when appended to the months.

34. Required the interest of $962 84*ct.* for 5*y*. 4*m*. 22*d*. at 6 per cent.

```
                 $ ct.
               |½|962,84
                 ——————
Half the prin. 481,42
                64,73 Decimal of days.
               ———————
                    =31162,3166=Ans. $311 62ct. 3166m.
```

35. Required the interest of $576 77*ct.* for 4*y*. 5*m*. 26*d*. at 6 per cent.

```
        $ ct.
     |½|576,77
      ———————
      288,385
        53,86
      ———————
           =15532ct. 416m.  Ans. 155$ 32ct. 416m
      ———————
```

36. Required the interest of $125 75*ct.* for 11*m.* 17*d.* at 7 per cent.

```
           $ ct.
           125,75
           ------
           62,875     ct.  m.
            11,56=6|726,835=6  per cent.
           ------    121,139=1 per cent.
                    ---------
      Ans. $8,41,9.74 =7 per cent.
                    ---------
```

37. Required the interest of $313 28*ct.* for 2*y.* 7*m.* 28*d.* at 8 per cent.

```
        2|313,28
        --------      ct.  m.
       $ 156,64=3|5001,6152=6 per cent.
          31,93    1667,1717=2 per cent.
        --------   ---------
                   66,68,6869=8 per cent.
                   ----------
```

Ans. $66 68*ct.* 6869*m.*

38. Required the interest of $486 18*ct.* for 1*y.* 9*m.* 20*d.* at 5 per cent.

```
     2|486,18
     --------     ct.  m.
      243,09=6|5265,3294=6 per cent.
       21,66=    8775549=1 per cent.
     --------   ---------
                43,87,7645=5 per cent.
                ----------
```

Ans. $43 87*ct.* 7745 *m.*

39. Required the interest of $738 22*ct.* for 2*y.* 10*m.* 29*d.* at 7 per cent.

```
     2|738,22
                   ct.    m.
      369,11=6|12904,0856=6 per cent.
       34,96     2150,6809=1 per cent.
     --------   ----------
                150,54,7665=7 per cent.
                -----------
```

Ans. $150 54*ct.* 7665*m.*

To calculate interest by days, without any reference to years and months, and that *at any given rate per cent.*, with entire exactness.

Rule.

Multiply the given sum by the whole number of days, and divide the product by the following divisors, framed in conformity with the required rate of interest, viz.

For 4	per cent	- - - -	9120
— 5	do.	- - - -	7300
— 6	do.	- - - -	6083
— 7	do.	- - - -	5214
— 8	do.	- - - -	4562

These divisors are thus obtained. To 365 days annex two ciphers, which is equal to multiplying them by 100: this product divided by the given rate per cent. furnishes these respective divisors. Thus:—

$$\left.\begin{matrix}4\\5\\6\\7\\8\end{matrix}\right\} 36500 \left\{\begin{matrix}9120 \text{ for } 4\\7300 \text{ for } 5\\6083 \text{ for } 6\\5214 \text{ for } 7\\4562 \text{ for } 8\end{matrix}\right\} \text{per Cent.}$$

40. Required the interest of $210 28*ct.* for 65 days, at 7 per cent.

```
      $ ct.
     210,28
         65
     ------
     105140
    126168
    -------
5214)13668,20(
```

Ans. $2 62*ct.* 1*m.*+

41. Required the interest of $502 12*ct.* 5*m.* for 105 days, at 6 per cent.

```
      $  ct.
     502,125
         105
     -------
     2510625
    5021250
    --------
6083)52723125(
```

Ans. $8 66*ct.* 7*m.*+

42. Required the interest of \$463 44*ct.* for 88 days, at 5 per cent.

$	ct.
463,44	
88	
370752	
370752	

7300)40782,72(

Ans. \$5 58*ct.* 6*m.*+

43. Required the interest of \$628 78*ct.* for 208 days, at 4 per cent.

$	ct.
628,78	
208	
503024	
1257560	

9120)130786,24(

Ans. \$14 34*ct.* 1*m.*+

44. Required the interest of \$389 88*ct.* for 150 days, at 8 per cent.

$	ct.
389,88	
150	
1949400	
38988	

4562)5848200(

Ans. \$12 81*ct.* 9*m.*+

45. Required the interest of £228 15*s.* 9*d.* for 87 days, at 7 per cent.

£
228,7875
87

16015125
18303000

5214)19904,5125(

Ans. £3 16*s.* 4*d.* 2*qr.*

46. Required the interest of £523 8*s.* 6*d.* for 106 days, at 6 per cent.

£
523,425
106

3140550
5234250

6083)55483,050(

Ans. £9 2*s.* 5*d.*

47. Required the interest of £240 10*s.* for 23 days, at 5 per cent.

£
240,5
23

7215
4810

7300)5531,5(

Ans. £0 15*s.* 1*d.* 3*qr.* 39.

48. Required the interest of $167 50*ct.* for 42 days, at 6 per cent.

```
 $ ct.
 167,50
     42
 ------
  33500
 67000
 ------
6083)7035,00(
```

Ans. $1 15*ct.* 6*m.*

49. Required the interest of $401 55*ct.* for 44 days, at 7 per cent.

```
 $ ct.
 401,55
     44
 ------
 160620
160620
 ------
5214)17668,20(
```

Ans. $3 38*ct.* 8*m.*

An easy method of computing interest upon an account current, whereby the creditor may ascertain what would be his just remuneration for the use of his funds while in arrears.

A and *B* have an open cash account, in which *A* loans and receives of *B*, as follows:—

		dol. on int. d. pro. &c.
Sep. 25, 1827.	Lent	150×13=1950
Oct. 8, –	Lent	75
		225× 7=1575
– 15, –	Rec.	180
		45× 7= 315
– 22, –	Lent	89
		134×12=1608
Nov. 3, 1827.	Rec.	120
		14×17= 238
– 20, –	Lent	302
		316×10=3160
– 30, –	Rec.	280
		36×12= 432
Dec. 12, 1827.	Lent	250
		286× 8=2288
– 20, –	Lent	84
		370×11=4070

			370×11=4070		
Dec. 31,	-	Rec.	268		
			102×10=1020		
Jan. 10, 1828.		Lent	148		
			250× 9=2250		
- 19,	-	Rec.	243		
- 30.		Time of set.	7	127	18983

From September 25th, 1827,
To January 30th, 1828, 127

Total product of principal and time.

$$\left.\begin{matrix}7300\\6083\\5214\end{matrix}\right\}18983\left\{\begin{matrix}2,87,4\\3,10,4\\3,64,8\end{matrix}\right.$$

	$ ct. m.
	2 87 4
Principal	7
Amount	$9 87 4 *Ans.* 5 per cent.

	$ ct. m.
	3 10 4
Principal	7
Amount	$10 10 4 *Ans.* 6 per cent.

	$ ct. m.
	3 64 8
Principal	7
Amount	$10 64 8 *Ans.* 7 per cent.

The several amounts, at the different rates per cent., are here exhibited. Also the proof that the days correspond with the whole number of days in the given time.

Example 2d.—In which debit and credit are distinguished in the products of principal and time.

Date		*dol. on int.*	*d.*	*Dr.*	*Cr.*
Sep. 1, 1827.	Lent	300 ×	7	=2100	
— 8, —	Lent	400			
		+700 ×	14	=9800	
— 22, —	Rec.	900			
		—200 ×	8		=1600
— 30, —	Rec.	400			
		—600 ×	16		=9600
Oct. 16, 1827.	Lent	1000			
		+400 ×	12	=4800	
— 28, —	Lent	600			
		+1000 ×	8	=8000	
Nov. 5, 1827.	Rec.	800			
		+200 ×	10	=2000	
— 15, —	Rec.	900			
		—700 ×	15		=10500
— 30, —	Rec.	500			
		—1200 ×	10	——	12000
Dec. 10, 1827.	Lent	1300			
		+ 100 ×	7	= 700	
— 17, —	Lent	600			
		+700 ×	7	=4900	
— 24, —	Rec.	650			
— 31, —		50 ×	7	= 350	
Time of settle. from Sep. 1, 1827,			121	32650	33700
To Dec. 31, 1827 - - - - -			121	——	32650
Interest, and balance of Cr.					1050

7300		14ct. 3m.
6083	1050,000	17ct. 2m.
5214		20ct. 1m.

Principal. Dollars	50,000	
Deduct Interest	14,3	5 per cent.
Ans. $49	85 7	

Dollars	50,000	
	17,2	6 per cent.
Ans. $ 49	82 8	

Dollars	50,000	
	20,1	7 per cent.
Ans. $49	79 9	

Balance in favour of debit, as seen by the different rates of interest, when deducted from the 50 dollars, principal, due December 31st.

Note.—If the given time be months, multiply each sum by the months it is at interest; and take for a divisor the quotient of 1200; viz. 12 months multiplied by 100, divided by the rate per cent. required. Thus:—

5		240 5 per cent.	This is precisely on the
6	1200	200 6 per cent.	same principle as that of
7		171,4 7 per cent.	days.

To find the *principal*, when the amount, time, and rate per cent. are given.

Rule.

As the amount of 100 pounds or dollars, at the rate and time given, is to the amount given; so is 100 pounds or dollars, to the principal required.

1. What principal, at interest for 6 years, at 6 per cent. per annum, will amount to 544 dollars?

y. r.		dol.			
6×6=36	Interest of 100 for 6 years.				
100					
		dol.	dol.	dol.	
136	Then, as 136	: 544	: : 100	:	Ans. 400 Dol.

136)54400(400
544
00

2. What principal, at interest for 4 years, at 7 per cent. will amount to 832 dollars?

4×7= 28					
100					
	dol.	dol.	dol.		dol.
128	As 128	: 832	: : 100		83200÷128=650 Ans.

3. What principal, at interest for 8 years, at 5 per cent. will amount to 1120 dollars?

= 40					
100					
	dol.	dol.	dol.		dol.
140	As : 140	: 1120	: : 100	:	Ans. 800.

To find the *rate* per cent., when the amount, time, and principal are given.

RULE.

As the product of the time and principal is to 100 pounds or dollars; so is the interest of the whole time to the rate per cent.

1. At what rate per cent. per annum, will 400 dollars amount to 544 dollars, in 6 years?

			dol.	dol.	dol.
400 Principal.	544 Amount.		As 2400	: 100	: : 144 :
6 Years.	400 Principal.				
		2400)14400(6	Ans. 6 per cent.		
2400	144 Interest.	14400			

2. At what rate per cent. per annum, will 650 dollars amount to 832 dollars, in 4 years?

	dol.		dol.		dol.	
As	2600	:	100	: :	182	*Ans.* 7 per cent.

3. At what rate per cent. per annum, will 800 dollars amount to 1120 dollars, in 8 years?

	dol.		dol.		dol.		
As	6400	:	100	: :	320	:	*Ans.* 5 per cent.

To find the *time*, when the principal, amount, and rate per cent. are given.

RULE.

As the interest of the principal for one year, is to the whole interest; so is one year to the time required.

1. In what time will 400 dollars amount to 544 dollars, at 6 per cent. per annum?

dol.		dol.		dol.		yr.		yr.
400×6=24|00	As	24	:	144	: :	1	144÷24=6	*Ans.* 6.

2. In what time will 650 dollars amount to 832 dollars, at 7 per cent. per annum?

dol.		dol.		dol.		yr.
650×7=45,50	As	45,50	:	182,00	: :	1

182,00÷45,50=4 *Ans.* 4 years.

3. In what time will 800 dollars amount to 1120 dollars, at 5 per cent. per annum?

	dol.		dol.		yr.		
As	40	:	320	: :	1	:	*Ans.* 8 years.

4. A legacy was left to a child of 2500 dollars, the amount of which, at the rate of 6 per cent. per annum, he would receive when he should arrive to the age of 21 years. At the age required, he found the amount to be 4000 dollars. How old was he when the legacy was granted? *Ans.* 11 *years.*

COMMISSION AND BROKERAGE.

Commission and brokerage are compensations to factors and brokers, for their services rendered.

Rule.

Find the interest of the given sum, as if for one year, at the rate proposed. Should the rate be less than one per cent., then take such aliquot part or parts of the interest of 1 per cent., as the required rate is of a pound, or a dollar.

1. What is the commission on £6521 5*s*. 6*d*. at 6 per cent.?

```
  £   s.  d.
 652 15   6
          6
 ------------
39|16 13  0
    20
   ---
  3|33
     12
    ---
  3|96
     4
    ---
  3|84
```

The same result is obtained by dividing the given sum by the aliquot parts of the rate per cent. Thus:—

```
 £        £   s.  d
|5|1/20|652  15   6
        ----------------
|1|1/5 | 32  12   9,1,2
          6  10   6 2,64
        ----------------
       £39    3   3 3,84
```

Ans. £39 3*s*. 3*d*. 3*qr*.

2. What is the commission on 1275 dollars, at 5 per cent.?

dol. *dol. ct.* *dol. ct.* *dol. ct.*
1275×5=63,75; or 1275,00÷20=63,75 *Ans.* 63 75.

3. What is the commission on 1826*dol*. 75*ct*. at 3½ per cent.?
Ans. 63*dol*. 93*ct*. 6½*m*.

4. What is the brokerage on 1650 dollars, at $1\frac{1}{2}$ per cent.?
Ans. 24*dol.* 75*ct.*

5. What is the brokerage on £434 12*s.* 6*d.* at $\frac{3}{4}$ per cent.?
Ans. £3 5*s.* 4*d.* 1*qr.*

6. What is the brokerage on £3675 14*s.* 2*d.* at 5*s.* per cent.?
Ans. £9 3*s.* 9*d.* 1*qr.*

7. What is the commission on 2674*dol.* 25*ct.* at 15 cents, per cent.? *Ans.* 4*dol.* 1*ct.* 1*m.* 375.

8. A factor receives 1200 dollars to lay out: after deducting his commission of 5 per cent., how much will remain to be laid out?

	dol.	*dol.*	*dol.*	*dol. ct. m.*
As	100×5 :	100 : :	1200 :	*Ans.* 1142 85 7+

9. What is the brokerage on 2968*dol.* 34*ct.* at $\frac{7}{8}$ per cent.?

```
 dol. ct.
 2968,34
       7
 --------
8)20778,38
 --------
 25,97,2975    Ans. 25dol. 97ct. 2m.+
```

10. What is the commission on £2136 14*s.* 4*d.* at $\frac{3}{4}$ per cent.?
3÷4 *Ans.* £16 0*s.* 6*d.*

11. What is the brokerage on 2846*dol.* 48*ct.* at $\frac{1}{4}$ per cent.?
Ans. 7*dol.* 11*ct.* 6*m.* 2.

12. Required the net proceeds of certain goods, amounting to £762 10*s.* allowing a commission of $2\frac{1}{2}$ per cent.
Ans. £19. 1*s.* 3*d.*

INSURANCE.

Insurance is an exemption from hazard, by paying, or securing a certain sum, on condition of being indemnified for loss or damage.

Policy is the name given to the instrument, by which the contract of indemnity is effected between the insurer and insured.

Rule.—The method of operation is the same as in interest.

1. What is the premium of insurance on £1233 6*s.* 8*d.* at 8 per cent.? *Ans.* £98 13*s.* 4*d.*

2. What is the premium on 2500 dollars, at 10 per cent.? *Ans.* 250 dollars.

3. What is to be received for a policy of 3000 dollars, deducting a premium of 15 per cent.? *Ans.* 2550 dollars.

4. What sum must a policy be taken out for, to cover 2112 dollars, when the premium is 12 per cent.?

100 Policy.		As $ 88	: $ 100	: : $ 2112 :	*Ans.* $ 2400.
12 Premium.					
88 Sum covered.		Proof $2400 at 12 per cent.			
		12		2400 Policy.	
		288 : 00		288 Premium.	
		Sum covered		2112 *Ans.*	

5. What sum must a policy be taken out for, to cover 1800 dollars, when the premium is 10 per cent.? *Ans.* 2000*dol.*

6. What is the premium of insuring 1250 dollars, at 7½ per cent.? *Ans.* 93*dol.* 75*ct.*

BUYING AND SELLING STOCKS.

Stock, in the sense here used, is a fund established, either by government or by individuals in a corporate capacity, the value of which is variable.

The rule is similar to that of interest.

1. What is the amount of 3376 dollars, bank stock, at 20 per cent. advance?

3376+675,20= *Ans.* 4051*dol.* 20*ct.*

2. What is the amount of $1645, national bank stock, at 132 per cent.?

```
     $
  1645
   132  Multiply by the per cent.
  ----
  3290
 4935
1645
-------
2171,40          Ans $2171 40ct.
```

What is the amount of $2736, bank stock, at 25 per cent. advance?

```
     $                Or thus:—
  2736
   125                        $
  -----              |25|¼|2736
  13680                     684
 32832                     ----
 --------                  3420
 $3420,00                  ----
                        Ans. $3420.
```

4. What is the amount of 4264 dollars, deferred stock, at 89 per cent.?

4264 × 89 = *Ans.* 3794*dol.* 96*ct.*

5. What is the amount of 1818 dollars, State notes, at 83½ per cent.? *Ans.* 1518*dol.* 3*ct.*

6. What is the amount of 2856 dollars, of 3 per cent. stock, at 56¾ per cent.?

2856 × 56¾ = *Ans.* 1620*dol.* 78*ct.*

COMPOUND INTEREST.

Compound Interest is that which arises from any principal and its interest being put together, when the interest becomes due and is not paid; and consequently it draws interest upon interest: it is hence called Compound Interest.

RULE.

Find, by Simple Interest, the amount of the given sum for the first year, and this amount becomes the principal for the second year; then find the amount of this last principal for the second year, and it becomes the principal for the third year; and thus proceed for any number of years required.

From the last amount subtract the given principal, and the remainder will be the Compound Interest.

1. What is the Compound Interest of \$284 26*ct.* for 2 years, at 6 per cent.?

\$ *ct.*	
284,26	
6	rate per ct.
17,05.56	int. 1 year.
284,26	prin. add.
301,31.56	amt. 1 yr.
6	rate per ct.
18,07.8936	int. 2d. yr.
301,31.56	prin. add.
\$ 319,39.4536	amt. 2 yrs.

Or thus:—

		\$ *ct.*	
5	$\frac{1}{20}$	284,26	
1	$\frac{1}{5}$	14,213	
		2,8426	
5	$\frac{1}{20}$	301,31.56	amt.
1	$\frac{1}{5}$	15,06.578	1 yr.
		3,01.3156	
		319,39.4536	
Prin. deduct.		284,26	
Com. Inter.		\$ 35,13.4536	

Ans. \$35 13*ct.* 4*m.* 536.

2. How much is the Compound Interest of £346 12*s.* 6*d.* for 3 years, at 6 per cent.?

£ *s.* *d.*
346 12 6
6
20|79 15 0
20
15|95
12
11|40
4
1|60

346 12 6	
20 15 11 1	
367 8 5 1	Amt. 1 yr.
6	
22|04 10 7 2	
20	
90	
12	
10|87	
4	
3|50	

£	s.	d.	qr.	
367	8	5	1	
22	0	10	3	
£389	9	4	0	Amount, 2d year.

£	s.	d.
389	9	4
		6
23,36	16	0
20		
7|36		
12		
4|32		
4		
1|28		

£	s.	d.	qr.	
389	9	4		
23	7	4	1	
£412	16	8	1	Amt. 3d yr.

£	s.	d.	qr.	
412	16	8	1	Amt. 3.
346	12	6		Princi.
£ 66	4	2	1	Com. Int.

[*Another Method.*]

£		£	s.	d.	qr.
5	1/20	346	12	6	
1	1/5	17	6	7	2
		3	9	3	3
5	1/20	367	8	5	1
1	1/5	18	7	5	
		3	13	5	3
		£389	9	4	0

£		£	s.	d.	qr.	
5	1/20	389	9	4		
1	1/5	19	9	5	2	
		3	17	10	3	
		412	16	8	1	Amt.
		346	12	6		Prin.
		£ 66	4	2	1	Com. Int.

3. What is the amount of $1242, for 6 years, at 6 per cent., Compound Interest? *Ans.* $1761 78*ct.* 9*m.*+

A table is often used to facilitate the labour, in obtaining answers to questions in Compound Interest. It is so constructed, as to exhibit the amount of one pound, or one dollar, for any number of years up to 50, at the rates of 4, 5, and 6 per cent. per annum, at Compound Interest. This table will be seen at

or near the close of this book, under the title of "Table I., showing the amount of $1, or £1, at Compound Interest." It is thus constructed:—

To unity or 1, add ,04 decimally for four per cent.; ,05 for 5 per cent.; and ,06 for 6 per cent.; and in a similar manner for any other rate per cent. The significant figures of the decimal are always expressive of the rate per cent. required. The reason why these different rates per cent. are added to 1, is the following. The 1,04 will be the tabular number for one year, at 4 per cent.; for if the interest be 4 cents on the dollar, or 4 hundredths on a pound, for one year, then it will give $4 on $100, or £4 on £100, annually: and consequently the ratio will be the same, whether the given principal be larger or smaller. This tabular number, therefore, multiplied decimally by the given principal, whatever it may be, will give the amount for one year for that principal. For example:

1,04×500 dollars, is equal to $520,00, for 1 year, at 4 per cent.
1,05×500 dollars, is equal to $525,00, for 1 year, at 5 per cent.
1,06×500 dollars, is equal to $530,00, for 1 year, at 6 per cent.

The tabular number for the second year is found by multiplying 1,04 by 1,04, and 1,05 by 1,05, &c. for 4, 5, and 6 per cent. respectively.

The tabular number of each successive year, up to any given number of years, may be obtained by multiplying the tabular number of the last year by the tabular number of one year. Or, if the tabular number of the second year be multiplied by itself, it will give the tabular number for 4 years: or the tabular number of 3 years be multiplied into itself, it will give the tabular number for 6 years: or the tabular number of 4 years involved into itself will give the tabular number for 8 years; and so for whatever rate per cent. the table shall be constructed. If for 5 years, multiply the tabular number of 4 years by that of one year.

It will be observed, that this process is on the principle of geometrical progression, because the interest required is compound; whereas simple interest is founded on the principle of arithmetical progression. Hence, the multiplying of the tabular

number for 3 years into itself does not produce the square of 3 =9, but simply the double of 3, viz. 6 : so of 4 years into itself, the square of 4 is not produced, but it is merely doubled, furnishing the tabular for 8 years. The same would be true, whatever the rate per cent. in Compound Interest.

∴ By either method, therefore, the tabular numbers may be constructed, and both with entire exactness.

It will also be observed, that 6 decimal numbers are sufficiently accurate, to be used in the multiplications; as 2 or 3 of the left-hand decimals express the principal value.

Because in the table, 6 places of decimals are observable, as against 1 year, in the 4, 5, and 6 per cent. tabulars, the 4 last of which places are ciphers; these ciphers are not added on account of any value; for in their use they possess none, but merely to fill the vacancy, so as to produce uniformity in numbers with the succeeding tabulars.

If it is difficult to tell a short story, it is important to make it *plain*, in order to render it intelligible.

To apply this table in computing Compound Interest, multiply the tabular standing against the given number of years, and under the rate per cent. required, by the given principal, and the product is the amount sought. The figures on the left of the separatrix are pounds or dollars, as are specified in the question; if in dollars, the first two figures at the right are cents, and the third mills; if in pounds, the figures at the right are decimals of a pound.

4. What is the amount of $800, for 4 years, at 6 per cent. Compound Interest?

The tabular number for 4 years, at 6 per cent., is

	1,262477	
	800 given principal.	
Amount	1009,981600	*Ans.* $1009 98*ct.* 1*m.* 6.

5. What is the amount of $800, for 4 years, at 5 per cent. Compound Interest? *Ans.* $972 40*ct.* 4*m.* 8.

6. What is the amount of $800, for 4 years, at 4 per cent. Compound Interest? *Ans.* $935 88*ct.* 7*m.* 2.

7. What is the amount of $2150, for 15 years, at 6 per cent. Compound Interest? *Ans.* $5152 59*ct.* 9*m.* 7.

8. What is the amount of the above sum, and time also, at 4 per cent. Compound Interest? *Ans.* $3872 02*ct.* 9*m.* 6.

9. What is the amount of £890 10*s.* for 6 years, at 6 per cent. Compound Interest?

Tabular 1,418519×£890,5= *Ans.* £1263 3*s.* 9*d.* 3*qr.*+

10. What is the amount of $2350, for 10 years, at 5 per cent. Compound Interest? *Ans.* $3827 90*ct.* 3*m.*+

11. On examining some old school bills, one of $12, due for tuition in January, 1800, and not yet cancelled: [ah, would there were no others of a similar description!] what is the just amount of this old claim, now January, 1828, at 6 per cent. per annum, Compound Interest?

The tabular number for 28 years, at 6 per cent. is
5,111687×12*dol.*= *Ans.* $61 34*ct.*+

12. What is the Compound Interest of $950, for 3 years, at 7 per cent. Compound Interest? *Ans.* $213 79*ct.*+

DISCOUNT.

Discount is a deduction or allowance made for the receipt of money before it becomes due. The allowance made is such, that the remainder, or sum received, if put at interest for the same time, and at the same rate, would amount to the original sum.

This rule is applicable more particularly to notes, bills, and obligations, due at a future period, but not on interest. An obligation on interest, payable at a future time, by ready payment, ceases to draw interest. Thus; if $100, at 6 per cent.

would cancel a debt of \$106 at the expiration of one year, then the creditor might safely receive the \$100 as a present payment; for the \$100 immediately put to interest again, at 6 per cent., would, at the close of the year, amount to the same as the original claim. Hence, what remains after the discount is made, is the present worth.

There are two rules to be applied in discount; the one to find what the discount is; the other to find the present worth.

To find the discount:

Rule.

As the amount of £100 or \$100, at the rate and time given, is to the interest of 100, at the same rate and time, so is the given sum to the discount.

Subtract the discount from the given sum, and the remainder is the present worth. Or to find the present worth:

Rule.

As the amount of 100, at the given rate and time, is to 100, so is the given sum to the present worth.

Proof.—Find the amount of the present worth, at the given rate and time, and if the work is right, that will be equal to the given sum.

1. What is the discount of 575*dol.* 50*ct.* for 16 months, at 6 per cent. per annum?

		dol. *dol.* *dol.* *ct.*
6 per cent. 16*m.*=8+100.		As 108 : 8 : : 575,50

Then 575,50×8÷108=\$42 62*ct.* 9*m.* 6.

575,50	Principal.
42,62.96	Discount.
\$532,87.04	Present worth.

2. What is the present worth of £562 18*s.* 6*d.* for 10 months, at 6 per cent.?

	£	£	£	£ *s.* *d.* *qr.*
As	105 :	100 : :	562,925 :	*Ans.* 536 2 4 2.+

3. What is the present worth of 450 dollars, due in 18 months, at 6 per cent.?

100*dol.* 18*m.* at 6 per cent. will give 9*dol.*, which added to 100*dol.* gives the amount 109*dol.* Then for the present worth:

dol.	*dol.*	*dol.*		*dol. ct. m*
As 109 :	100 : :	450	Then 450×100÷109=*Ans.*	412 84 4.

Proof.
450, principal.
412,84.4 present worth.

37,15.6 discount.

Or thus:— $412,84.4
6 rate per cent.

½|2477,068 interest for 1 year.
1238,534 interest for ½ a year.

37,15.602 amount of interest.
412,84.4 present worth.

$450,00.002

4. What is the discount of £500, due 4 years hence, at 5 per cent.?

£	£	£	£ s. d.
As 120 :	20 :	500 :	*Ans.* 83 6 8.

5. Bought goods to the value of $1136 28*ct.* on a credit of 9 months; what present payment will discharge the same, allowing 6 per cent. discount? *Ans.* $1087 35*ct.*

6. What is the difference between the interest of $1630, at 7 per cent. per annum, for 7 years, and the discount of the same sum for the same time and rate per cent.? *Ans.* $262 66*ct.*

9. What is the present worth of $1000, ¼ payable in 3 months, ¼ payable in 6 months, and the remainder at the end of the year, at 7 per cent. discount?

	m.		$ *ct.*		$ *ct. m.*
Equated time	8,25	Discount	45,91.53	*Ans.*	954,08.47.

Note.—It has already been remarked, that discount is parti-

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

cent., notes of 30, 60, or 90, &c. days, ever include in their respective discounts, three additional days, called days of grace, for which discount is taken; viz. for 30 days, 33; for 60, 63; and for 90, 93 days. Although these several notes would fall due at the expiration of the periods severally specified, yet if paid within the 3 days of grace, the notes are not dishonoured.

To find the discount of any given sum, for 33, 63, or 93 days, it has been usual to take one-sixth of the days for a multiplier, and multiply it into the given sum, and the product will be mills; or cut off the right-hand figure of the product, as one figure in the multiplier is a decimal, and the residue of the product is cents.

1. To find the discount of 100 dollars, for 30, 60, or 90 days, at 6 per cent.

6|33	6|63	6|93
,5½	1,0½	1,5½
100	100	100
,5½	1,0½	1,5½
50	50	50
500	1000	500
		100
Cents. ,55.0	$1,05.0	$1,55.0

2. To find the present worth, or net proceeds, of 500 dollars, for 30, 60, or 90 days, at 6 per cent.

dol.	*dol.*	*dol.*
500	500	500
,5½	1,0½	1,5½
250	250	250
2500	5000	2500
		500
Dol. 2,75.0	Dollars 5,25.0	Dollars 7,75.0
Dollars 500	Dollars 500	Dollars 500
2,75	5,25	7,75
Dollars 497,25	Dollars 494,75	Dollars 492,25
For 30.	For 60,	For 90 days

The foregoing method of calculating bank discount has been given, as it has been adopted by general practice, in banking institutions, until a recent date. But having been discovered to be erroneous, it is now chiefly, if not universally abandoned. On this plan, 360 days constituted one year; consequently 30 days were reckoned one month, 60 days two months, and 90 days three months, &c. In this manner, 5 days were lost in every year, and 6 days in every fourth or leap year.

To correct this error, interest tables are now calculated at 6 per cent., the established rate per cent. for bank discount, calling three years out of four, 365 days, and every fourth year 366 days. When large loans are obtained from banks, the daily discount is very considerable. On a sum equal to that for which one check was given on a bank in this city, not long since, viz. half a million of dollars, the daily discount would exceed 80 dollars; and the 6 extra days in a year would amount to about 500 dollars. Thirty days are the exact twelfth part of 360; for $30 \times 12 = 360$; but it requires 30 days and 10 hours to produce the 12th part of 365 days. This difference is indeed small, being less than the half of a day; and parts of days not being brought into account, common usage has established the custom of making the general estimate of 30 days to one month. As the interest on large sums would necessarily amount to some dollars hourly, it is but just and equitable to adopt accurate rules for the computation of discount *for days*, at least. The three days' grace, which are appended to notes discounted at banks, are justly included in the discount, but fall proportionally short of the true time, in the same ratio of 30 days to 365.

The following method will prevent all embarrassments on this subject, and insure accuracy in the result.

Rule.—Multiply the number of days specified in the note, including the 3 days grace, by the number of dollars for which the note is given; and divide the product by 6083, and the quotient will be the discount at 6 per cent.

The method of obtaining the divisor was given on the 123d page. It may be proper here to repeat it, to render the fallacy of the old plan more conspicuous. To 365 days annex two ciphers, which is multiplying them by 100, and divide the pro-

duct by 6, the rate, and it gives the quotient 6083. For l year take 366 days, and proceed as before, and it gives quotient 6100. The latter quotient is the proper divisor leap year; and the former for other years. Or, if tables are ready calculated for 365 days, dividing the tabular number 365, and adding the quotient to that tabular number, the amo would be the discount for leap year.

The error in the old plan, arose from taking 360 days for a year; adding the two ciphers, and dividing by 6, gave divisor of 6000.

This will clearly appear by the following examples :—

To find the interest of 100 dollars for one year, by the method first. The product of the dollars and days gives 365 which divided by 6000=$6,083. On the new method, 365 ÷6083=$6,00. (It will be noticed, that the last figure in the divisor ,3, is not an exact third, and therefore two are added to its product, to produce the cipher, and restore the deficiency of the third.) For leap year, 36600÷6100=$6,00.

It is thus evident, that by the former method, 100 dollars for one year, draws 8 cents and 3 mills too much, and has consequently been pronounced to be usurious. The latter method proves the work correct.

But should it be urged, that this exhibits proof that there is no radical difference between Discount and Interest; let it be remembered, banks are authorized to receive their premiums or discounts, on notes *in advance;* and although computed at the rate of 6 per cent., yet is in reality more, operating as Compound Interest. It is evident therefore, that the radical difference between Discount and Interest is strictly maintained. It was doubtless this consideration, which has caused the blending of these distinct principles, as if in perfect unison, or entirely synonymous; whereas the distinction is clearly manifest, and highly important in its consequences.

In the above examples, 100 dollars for one year was designedly taken, more clearly to illustrate the fallacy of the former, and the correctness of the latter method of finding the true discounts. But it will be observed, that a bank note for one year will draw a premium for 368 days for common years; and 369

days for leap year, the three days' grace being added to the year. The real bank discounts, therefore, for one year, by the different methods, will be the following. On the old method, 36800÷6000=\$6,133:—on the modern plan, for a common year, 36800÷6083=\$6,049:—for leap year, 36900÷6100=\$6,049. Here a surplus is found again, as before, of 8 cents and 4 mills, by the former method.

The rule therefore last given will furnish the correct bank discount for any sum and given time; and by bestowing some labour, tables may readily be constructed upon this principle, which will greatly facilitate banking operations.

Note.—Discount, without any reference to time, is assimilated to Interest, Commissions, Brokerage, &c.: but when time is to be considered, or is necessarily implied, Discount differs essentially from Interest.

Questions relative to Interest, Commissions, Insurance, Discount, &c.

1. How many kinds of Interest are there?
2. What is Simple Interest?
3. What is the rate per cent. established by law?
4. What is meant by principal?
5. What is meant by rate, or interest?
6. What is meant by amount?
7. What is meant by per cent. and per annum?
8. For what other calculations are the rules of Interest applicable?
9. How is the Interest of any given sum found for one year?
10. Upon what principle is this rule founded?
11. What is the rule of Interest a contraction of, and how does it appear?
12. Were the questions stated at full length, to what rule would it strictly belong; and why?
13. Does multiplying the given principal by the rate and dividing by 100, clearly preclude a full statement and operation in the Single or Double Rule of Three?
14. Why is 100 made the first term, or what is the same, cutting off the two right-hand figures of the product?

15. What is the rule for computing the interest in Federal Money?

16. What is the rule when the principal is given in cents only?

17. What is the rule, when the given principal is pounds, shillings, and pence, of any particular currency, and the interest is required in Federal Money?

18. What is the reason for this rule?

19. What is the rule for finding the Simple Interest of any sum, for any given number of years, and parts of years?

20. Are 30 days to a month sufficiently exact, when computing interest on very large sums?

21. What rule, in such cases, is best calculated for days?

22. What is the rule, when the given time is an even number of months?

23. Why does multiplying by half the number of months give 6 per cent.?

24. What is the rule when the given months are odd; and why?

25. When the given time is months, what is the rule to find the interest, at *any given rate per cent.*; and why?

26. When the time is *months and days*, and the interest 6 per cent.; what is the rule to find the interest by *one operation*; and why?

27. How may any other rate per cent. be readily obtained from the 6 per cent.?

28. How may interest be calculated by days only, excluding the terms years and months, and that with exactness, at any given rate per cent.?

29. How are these different divisors constructed, and why?

30. What is the rule on a running account between two individuals, to compute the interest on funds while in arrears, for days?

31. If this running account be by months instead of days, what is the rule, and how is the divisor obtained?

32. What is the rule to find the *principal*, when the amount, time, and rate per cent. are given; and why?

33. What is the rule to find the *rate* per cent., when the amount, time, and principal are given?

34. What is the rule to find the *time*, when the principal, amount, and rate per cent. are given?

35. What is the method, in most common use, to compute interest on notes, &c. where there are partial endorsements?

36. What is the method established in Massachusetts, and generally adopted in the State of New-York, in similar cases?

37. What is the rule established in Connecticut?

38. What are Commission and Brokerage?

39. What is the rule for finding Commission and Brokerage?

40. What is Insurance? What is meant by Policy?

41. What is the method of computing the premium on Insurance?

42. What is meant by stocks; and are they subject to a variable value?

43. How are commissions calculated, for buying and selling stocks?

44. What is Compound Interest?

45. What is the rule for computing Compound Interest?

46. How is the Compound Interest found?

47. How is the amount found by the division of aliquot parts; and why?

48. Is Compound Interest more readily obtained by the aid of tabular numbers?

49. How are these tabular numbers constructed?

50. What is Discount? and on what principle is it founded?

51. To what is Discount more particularly applicable?

52. Are Interest and Discount the same, when time is considered?

53. How are they when time is not considered?

54. What is the rule, in Discount, to find the present worth?

55. What is the rule to find the discount?

56. What was the rule formerly practised to obtain bank interest?

57. Are three days' grace ever appended to notes for bank discount, and interest always taken on the same?

58. Did this method of computing bank interest prove to be correct?

59. Wherein did the error consist?

60. When the given sum was multiplied by the number of days, and that product divided by 6000; of how many days would the year consist?

61. What is the rule now adopted to find bank discount?

62. If the given sum be multiplied by the number of days, and that product be divided by 6083, of how many days would the year consist?

63. What will be the proper divisor, to find bank discount for leap year?

64. How is this divisor obtained?

EQUATION OF PAYMENTS.

The object of this rule is to find a mean time to pay at once several sums due at different times, so that no loss shall be sustained by either party.

Rule.

Multiply each payment by its time, and divide the amount the several products by the whole debt, and the quotient will be the mean time for the payment of the whole.

Examples.

1. *A* is indebted to *B* £400, of which one quarter is to be paid in 4 months; one quarter in 6 months; and the residue in 9 months. What is the mean or equated time for the payment of the whole sum?

100×4= 400
100×6= 600
200×9=1800

400 400)2800(7
2800

Ans. 7 months.

2. *C* sells *D* a farm for £800 to pay as follows:—£200 at 6 months; £200 at 12 months; £200 at 18 months; and £200 at 2 years. What is the equated time for the payment of the whole debt?

200× 6=1200
200×12=2400
200×18=3600
200×24=4800

£800 8,00)120,00(15
8
40
40

Ans. 15 months.

3. *E* is indebted to *F*, $1200, to be paid as follows:—$300 at 4 months; $600 at 8 months; and $300 at 1 year. What will be the equated time for the payment of the whole?

$$\begin{array}{l} 300\times\ 4=1200 \\ 600\times\ 8=4800 \\ 300\times12=3600 \\ \hline 1200\quad 12{,}00)96{,}00(8 \\ \qquad\qquad\quad 96 \end{array}$$

Ans. 8 months.

4. *G* purchases goods of *H*, to the amount of $1600, on the following conditions:—$400 to be paid down; $400 at the expiration of 8 months; and the remainder at 16 months. At what equated time may the whole be paid, so that neither party sustain any loss?

$$\begin{array}{l} 400\times\ 0=\ \ 400 \\ 400\times\ 8=\ 3200 \\ 800\times16=12800 \\ \hline 1600\ 16{,}00)164{,}00(10{,}25 \\ \qquad\qquad\quad 16 \\ \qquad\qquad\quad\ \ 40 \\ \qquad\qquad\quad\ \ 32 \\ \qquad\qquad\quad\ \ 80 \\ \qquad\qquad\quad\ \ 80 \end{array}$$

Ans. $10\frac{1}{4}$ months.

5. *I* sells *J* his share of a vessel for $750, on condition of remittances, as follow:—$150 in 30 days; $200 in 60; $200 in 90; and $200 in 120 days. At what equated time may *J* pay the whole sum?

$$\begin{array}{l} 150\times\ 30=\ 4500 \\ 200\times\ 60=12000 \\ 200\times\ 90=18000 \\ 200\times120=24000 \\ \hline 750\quad 750)58500(78 \\ \qquad\qquad\ \ 5250 \\ \qquad\qquad\ \ 6000 \\ \qquad\qquad\ \ 6000 \end{array}$$

Ans. 78 days.

6. *K* is indebted to *L* the amount of $1500, which falls due at the expiration of 4 months. *L* being pressed for money, receives at the expiration of 3 months $1200. What period of time ought *K* in equity to retain the remainder in his own hand?

$$\begin{array}{l} 1500\times4=6000 \\ 1200\times3=3600 \\ \hline 300\ 300)2400(8 \\ \qquad\quad\ \ 2400 \end{array}$$

Ans. 8 months.

7. *M* engages to pay to *N* $1000; half in 90, and the other half in 180 days. It being convenient to advance $750 at the expiration of 90 days, he is desirous of retaining the balance as much longer as a just equated time would allow him. How long may he defer the payment of the balance?—*Ans.* 270 days

8. *O* agreed to pay *P* $800, by installments of $50 each, once in two weeks, commencing payment at the end of the second week. What would be the equated time for the payment of the whole sum? *Ans.* 17 weeks.

BARTER.

BARTER is the exchanging of one commodity for another, according to the price or value agreed upon by the parties concerned.

RULE.

Find the value of the commodity whose quantity is given; then find what quantity of the other, at the proposed rate, can be bought for the same money; and the answer is obtained.

Examples.

1. How many pounds of butter, at 18 cents per pound, must be given in barter for 60 gallons of molasses, at 29 cents per gallon?

gal.	*ct.*	*\$ ct.*		*ct.*	*lb.*	*\$ ct.*	*lb.*	*oz.*
60×	29=	17,40	Then, as	18 :	1 : :	17,40 : *Ans.*	96	10.+

2. How many bushels of wheat, at $1 12½*ct.* per bushel, must be given for 15*cwt.* 2*qr.* 14*lb.* of sugar, at $11 20*ct.* per cwt.? *Ans.* 155*bu.* 2*pk.* 1*qr.* 1*pt.* 2*gl.*+

3. How many pounds of tea, at 72*ct.* per pound, must be given for 60 bushels of barley, at 84*ct.* per bushel?—*Ans.* 70 tea.

4. A farmer brought into market, 86 barrels of cider, which sold at $1 50*ct.* per barrel; for which he received $78 in cash; and the balance he took in tea, at 75 cents per pound, and sugar at 10 cents per pound, and each an equal number of pounds. How many pounds of tea and sugar did he receive? *Ans.* 60*lb.* of each.

5. Agricola sold in market, 80 bushels of rye, at 62 cents per bushel; 100 bushels of corn, at 65 cents; 50 bushels of oats, at 30 cents; for which he received $40 in money; 60 bushels of salt, at 60 cents per bushel; 10 quintals of fish, at $2 50*ct.* per quintal; 1*cwt.* of sugar, at $10 40*ct.* per cwt.; 5 gallons of brandy, at $1 25*ct.*; the balance he received in rice, at 4 cents per pound. How many pounds of rice did he receive?

Ans. 298¾*lb.*

6. *L* sold *W* 246 yards of linen, at 38 cents per yard, and received in payment 82 gallons of brandy. What did the brandy cost per gallon? *Ans.* 114*ct.*

7. A merchant has tea at 88 cents per pound, ready money, but in barter he will have $1; *B* has raisins worth 20 cents per pound. How must he rate his raisins per pound to equal the proportion in the value of the tea? *Ans.* 22*ct.* 7*m.*+

8. *A* has 2 tons of iron, at $5 per cwt.. *B* pays him $150 in money, and the balance in cheese, at 10 cents per pound, for it. How many pounds of cheese must *A* receive? *Ans.* 500*lb.*

9. *C* has 15 tierces of flax seed, at $8 a tierce, for which *D* gives him 22*cwt.* of rice, at $3 50*ct.* per cwt. and the residue in money. How much money must *C* receive? *Ans.* $43.

10. *D* has flour at $5 50*ct.* per barrel; but in bartering for iron, at $6 18¾*ct.* per cwt. he will rise 12½ per cent. on his price by the barrel. What must his price be by the barrel?

Ans. $6 18¾*ct.*

11. *W* and *Z* barter:—*W* has 100 bushels of wheat, at $1 10*ct.* per bushel, which he would exchange with *Z* for barley, at 82½*ct.* per bushel; but *W* claims in barter $1 30*ct.* the bushel for his wheat. How must *Z* sell his barley by the bushel to proportion it with *W's* bartering price, and how many bushels of barley will *W* receive?

Ans. *Z.'s* bartering price is 97½*ct.* and must give *W* 133⅓*bu.*

LOSS AND GAIN.

THIS is a rule which discovers what is gained or lost, in buying and selling goods; and teaches dealers to raise or fall their prices, so as to gain or lose so much per cent. or otherwise.

This rule is also founded on proportion; and the questions belonging to it are performed by the Rule of Three.

I. When buying and selling prices are given to find what is gained or lost by selling.

RULE.

1. Find the value of the commodity at the price it cost. Then,
2. Find its value at the price it sold for. The difference between these will be the gain or loss.

Or, as 1 yard, pound, &c. is to the quantity given; so is the gain or loss on one yard, pound, &c. to the whole gain or loss.

Examples.

1. A merchant sold 150 yards of silk, at $1,25*ct.* per yard, which cost him $1. How much did he gain by the sale?

	1,25		*yd.* *yd.* *ct.*
	1,00		As 1 : 150 : : 25 :
	——		150, 25÷1= *Ans.* $37 50*ct.*
Gain per yard	,25		

yd. $ *ct.* $ *ct.*
Or, 150×1,25=187,50
150× 0=150
——
37,50

2. Bought 15½*cwt.* of sugar, at $9 50*ct.* per cwt. and sold it for 10*ct.* a pound. What was the whole gain?

ct.	$ *ct.*	
15 ×4×2×38×10=	173,60	Amount of Sales.
15½×9,50 =	147,25	Purchase expense.
	——	
	$ 26,35	Gain.

II. To find what is gained or lost per cent.

Rule.

1. Find the gain or loss by subtraction: then,

As the prime cost is to 100; so is the gain or loss, to the gain or loss per cent.

Examples.

1. If Mercator buy broad cloth, at \$3 75*ct.* per yard, and sell it again at \$4 50*ct.* per yard; what does he gain per cent.; or in laying out 100 dollars?

	\$ *ct.*
Sold for	4,50
Cost	3,75
Gained per yard	,75

\$ *ct.* \$ *ct.*
As 3,75 : 100 : : 75

100×75÷375=

Ans. 20 per cent.

2. If *G* purchase oats, at 2*s.*, and sell them again at 2*s.* 8*d.* per bushel; what will he gain per cent.; or in laying out £100?

s. *d.* £
As 2 : 8 : : 100 : = *Ans.* £33⅓ per cent. or £33 6*s.* 8*d.*

III. To find at what rate goods must be sold to gain or lose so much per cent.

Rule.

As 100, whether pounds or dollars, is to 100 with the profit added, or loss subtracted; so is the prime cost to the gaining or losing price.

Examples.

1. If tea be bought at 56*ct.* per pound; how must it be sold by the pound to gain 12½ per cent.?

dol. *dol. ct.* *ct.*
As 100 : 112,5 : : 56 : *Ans.* 63 cents.

2. If tea be bought at 63*ct.* per pound; how must it be sold by the pound to lose 12½ per cent.?

dol. *dol. ct.* *ct.*
As 100 : 87,5 : : 63 *Ans.* 56 cents.

3. Bought 120 gallons of wine, at 1*dol.* 25*ct.* per gallon; by disaster 20 gallons leaked out; at what price must the residue be sold per gallon, to gain upon the whole prime cost, at the rate of 20 per cent.? *Ans.* 1*dol.* 80*ct.* per gallon.

IV. The gain or loss per cent. being known, to find what the goods cost.

Rule.

As 100, whether pounds or dollars, with the gain per cent. added, or the loss per cent. subtracted, are to 100; so is the selling price to the prime cost.

Examples.

1. If a bushel of wheat be sold for 1*dol.* 25*ct.* and there be gained 25 per cent.; what is the prime cost of the wheat?

	dol.	*dol.*	*dol. ct.*	*s.*
As	125 :	100 : :	1,25 :	*Ans.* 1*dol.*

2. If a yard of cloth be sold at 13*s.* and there be gained 30 per cent.; what is the prime cost of the cloth? *Ans.* 10*s.*

3. If a barrel of cider be sold for 1*dol.* 50*ct.* and there be gained 25 per cent.; what is the prime cost of the cider?

Ans. 1*dol.* 20*ct.*

V. When so much is gained or lost by selling goods at a certain rate, to find the gain or loss when sold at any other rate.

Rule.

As the first price is to the other price, so is 100 pounds or dollars, with the profit per cent. added, or the loss per cent. subtracted, to the gain or loss per cent.

Note.—If the answer exceed 100, the excess is gain per cent.: if it be less than 100, the deficiency is the loss per cent.

Examples.

1. If rye is sold at 6*s.* a bushel, and 20 per cent. is gained; what will be gained per cent. if sold at 7*s.* per bushel?

	s.	*s.*	£		£
As	6 :	7 : :	120 :	$120 \times 7 \div 6 = 140$	*Ans.* 40 per cent.

2. If wine is retailed at 1*dol.* 25*ct.* per gallon, and 25 per cent. is gained; what will be the gain or loss per cent. if it is sold at 95*ct.* per gallon?

dol. ct. dol. dol.
As 1,25 : 95 : : 125 : =95÷100 *Ans.* 5 per cent is lost.

3. If sugar be sold at 9*dol.* per cwt. and 10 per cent. is lost.; what will be the gain or loss per cent. if it is sold at 11*dol.* per cwt.?

dol. dol. dol.
As 9 : 11 : : 90 : =110. *Ans.* 10 per cent. gained.

PROMISCUOUS EXAMPLES.

1. Bought rye at 50*ct.* a bushel, and sold it at 25 per cent. loss; how much was it sold for per bushel? *Ans.* 37½*ct.*

2. If 500 bushels of barley be sold for 400*dol.* and gain 25 per cent. profit; what was the prime cost of a bushel?–*Ans.* 64*ct.*

3. If *A* sold a barrel of cider for 11*s.* 6*d.*, and gained at the rate of 15 per cent.; what would he have gained per cent. had he sold it at 12*s.*? *Ans.* 20 per cent.

4. If Mercator purchased 245 bushels of corn for 163*dol.* and sold it for 185*dol.* 05*ct.*; what was the profit on each bushel? *Ans.* 9*ct.*

5. How much per cent. is equal to 2*s.* 6*d.* on the pound? *Ans.* 12½ per cent.

6. At 16*ct.* profit on a dollar, how much is it per cent.? *Ans.* 16 per cent.

7. If a vintner buys 450 gallons of wine, at 7*s.* 6*d.* per gallon, ready money; and sells it immediately at 8*s.* the gallon, payable in 3 months; how much does he gain, if he allows 6 per cent. discount for the time, for present payment? *Ans.* £8 11*s.* 9*d.* 2*qr.* gain.

Note.—When goods are bought or sold on credit, the present worth of their price must be found by discount, in order to find the true gain or loss.

Example.—1. Bought 150 yards of cloth, at 12*s.* 6*d.* per yard, on 6 months' credit, and sold it again at 15*s.* per yard, ready

money; what is gained per cent., allowing 6 per cent. discount on the purchase? *Ans.* £23,5*s.* 5½*d.*

2. A distiller is offered 12000 bushels of rye, at 58*ct.* ready money, per bushel, or 60*ct.* with two months' credit: he prefers the ready payment price. How much will he gain by borrowing the money, at the rate of 7 per cent. per annum, and paying the cash price? *Ans.* He will gain 158*dol.* 80*ct.*

3. A broker sells a quantity of wheat, at 1*dol.* a bushel, and gains 25 per cent.: he afterwards sold of the same article to the the amount of 600*dol.* and gained 50 per cent. How many bushels were there in the last sale, and what was the price by the bushel; also, what was the prime cost by the bushel?

Ans. Last sale, 500*bu.* at 1*dol.* 20*ct.*; prime cost 80*ct.*

Questions relative to Equation of Payments, Barter, and Loss and Gain.

1. What is meant by Equation of Payments; and on what principle is it founded?
2. What is the rule?
3. What is Barter?
4. What is the rule?
5. What is meant by Loss and Gain?
6. When purchasing and selling prices are given, what is the rule to find what is gained or lost by selling?
7. What is the rule to find what is gained or lost per cent.?
8. What is the rule to find at what rate goods must be sold, to gain or lose so much per cent.?
9. The gain or loss per cent. being known, what is the rule to find what the goods cost?
10. When so much is gained or lost by selling goods at a certain rate, what is the rule to find the gain or loss, when sold at any other rate?

FELLOWSHIP.

Fellowship is a rule used by merchants and others, who have been connected in a joint partnership, by which they may

readily adjust their accounts, so that each may ascertain his share of the gain or loss, in proportion to his share of the joint stock.

It is by this rule, a bankrupt's estate may be divided among his creditors.

Fellowship is of two kinds, Single and Double; the latter is often called Compound.

Single Fellowship

Is when the several shares of joint stock are employed in business an *equal term* or length of time.

Note.—It is not to be understood, that the several *stocks* are equal; but merely the *term* of time they are connected together is *equal.*

Rule.

As the whole stock is to the whole gain or loss; so is each man's particular stock, to his particular share of the gain or loss. Or thus: as the whole stock is to each man's particular stock; so is the whole gain or loss to each man's share of the gain or loss.

Proof.—Add all the particular shares of gain or loss together, and if right, their sum will be equal to the whole gain or loss.

Examples.

1. *A*, *B*, and *C*, enter into a partnership in trade. *A* put in 1200*dol.*; *B* 1500*dol.*; and *C* 1800*dol.*; they gain in business 2000*dol.* What is each man's share of the profit?

A. *B.* *C.*
\$1200+\$1500+\$1800=\$4500 Amount of Stock.

	dol.		*dol.*		*dol.*			*dol. ct. m.*	
Then, as	4500	:	1200	: :	2000	:	*Ans.*	533,333	*A*'s.
	4500	:	1500	: :	2000	:		666,666	*B*'s.
	4500	:	1800	: :	2000	:		800,000	*C*'s.
							Proof	2000,000	

2. *D* and *E* purchase stock jointly to the amount of 2500*dol.*; of which *D* advanced 1600*dol.* and *E* 900*dol.*; but they lose by

the concern 1000*dol.* How is this to be apportioned between them agreeably to their stocks advanced?

	dol.	*dol.*	*dol.*		*dol.*	
As	2500 :	1000 : :	1600	*Ans.*	640	*D*'s loss.
	2500 :	1000 . :	900	-	360	*E*'s loss.
				Proof	1000	

3. *O* is indebted to *M* 800*dol.* to *N* 1000*dol.* and to *R* 600*dol.* but having failed, he has only 1600*dol.* to meet these claims, which he delivers to them, to be divided according to their respective demands. What is each one's share?

Ans.	*M*'s.	\$ 533,333=	Loss	\$ 266,667
	N's.	- 666,666=	-	" 333,334
	R's.	- 400,000=	-	" 200,000

4. A captain, mate, and twelve seamen took a prize worth 4000*dol.* of which the captain drew twelve shares; the mate four shares; and the sailors one share each. What did each draw?

Captain's shares	12
Mate's - - -	4
12 seamen, 1 each	12
	28 Shares.

sh.	*sh.*	*dol.*		*dol. m. ct.*	
28 :	12 : :	4000 :	*Ans.*	1714,286	Captain.
28 :	12 : :	4000 :	"	571,428	Mate.
28 :	12 : :	4000 :	"	1714,286	Each seaman.

5. Four persons make up a capital of 3000*dol.* for trade, by which they gain 1200*dol.* *A* put in 650*dol.* *B* 550*dol.* and *C* 850*dol.* How much was *D*'s stock, and what was each man's gain?

D's stock was 950 dollars.

A's share 260 *dol.*; *B*'s 220 *dol.*; *C*'s 340 *dol.*; *D*'s 380 *dol.* *Ans.*

6. Divide 1080 into 3 parts, which shall be to each other as 2, 3, and 4

Ans. 240+360+480=1080 Proof

7. Two merchants gained in trade 900*dol.*; and it was agreed in dividing the profits, that *A* should draw 5 times as much as *B*. What was the share of each?

Ans. *A*'s 750*dol.*; *B*'s 150*dol.*

8. Three persons are to share in a legacy of 2800*dol.* *A* is to take a certain sum; *B* twice as much; and *C* four times as much. What was each one's share?

Suppose *A*50+*B*100+*C*200=350 Shares.

Ans. *A*'s share 400*dol.*; *B*'s 800*dol.*; *C*'s 1600*dol.*=2800 Pr

9. *H* and *I* have gained in business 1650*dol.* of which *H* is to receive 20 per cent. more than *I*. What is the share of each?

Ans. *H* receives 900*dol.*; *I* 750*dol.*=1650 Proof.

10. A store, with its contents, valued at 9000*dol.* being burnt; of which $\frac{1}{3}$ belonged to *A*; $\frac{1}{2}$ to *B*; and $\frac{1}{6}$ to *C*. What loss will each sustain, allowing there was an insurance of 6000*dol.* upon it?

Ans. *A*'s loss 1000*dol.*; *B*'s 1500*dol*; *C*'s 500*dol.*

11. Three boys, *Richard*, *David*, and *Samuel*, purchased a lottery ticket for 5*dol.* for which *Richard* paid 1*dol.*; *David* 1$\frac{1}{2}$*dol.*; and *Samuel* 2$\frac{1}{2}$*dol.* This ticket drew a prize of 4000 dollars, subject to a deduction of 15 per cent. How much ought each to receive?

Ans. *Richard* 680*dol.*; *David* 1020*dol.*; *Samuel* 1700*dol.*

12. Three persons are entitled to the net proceeds of a lottery prize of 1800*dol.* *O* draws $\frac{1}{2}$, *P* $\frac{1}{4}$, and *Q* the remainder. But *Q* relinquishes his part to be divided between *O* and *P*, according to their own respective shares. What did *O* and *P* respectively receive? *Ans.* *O* received 1200*dol.*; *P* 600*dol.*

13. A bankrupt owed to sundry creditors the amount of 10,000 pounds; but on examination of his effects, he found that all he could raise would pay only 2*s.* 6*d.* on the pound. What was the amount of his effects? *Ans.* £1250.

14. Three dealers in a joint copartnership cleared 1800*dol.*; *A* had put in 800*dol.*; *B* 1000*dol.*; and *C* put in 4 horses, valued so high, as to entitle him to 720*dol.* of the profits. What were *C*'s horses estimated at, individually and collectively, and what were *A*'s and *B*'s share?

Ans. *C*'s horses were 300*dol.* each =1200	720
A's share - - - - - - - -	480
B's share - - - - - - - -	600
Proof	1800

DOUBLE; OR

COMPOUND FELLOWSHIP.

This rule relates to stocks which are employed in partnerships for unequal periods of time.

Rule.

1. Multiply each man's stock by the time it was continued in trade, and add the several products together.

Then say; as the sum of the several products is to the whole gain or loss; so is each man's particular product to his particular share of the gain or loss. Or thus:—

As the sum of the product is to each particular product; so is the whole gain or loss to its share of the gain or loss.

Examples.

1. Two merchants were united in trade. *A* put in £500 for 12 months; *B* put in £450 for 15 months, and gained £300. What is the share of each?

£ *m.*
500×12=6000 450×15=6750+6000=12750 Sum of the pro.

					£	s.	d.	qr.	
As 12750	: 600	: : 300	:	*Ans.*	141	3	6	$1\frac{325}{1275}$.	*A.*
12750	: 300	: : 6750	:	*Ans.*	158	16	5	$2\frac{750}{1275}$.	*B.*
				Proof.	£300	0	0	0	

2. Three persons enter into copartnership with a capital of $2400. *J* put in $900, 27 days; *L* $800, 24 days; *M* $700, 21 days. They clear $800. What is the share of each?

	$ ct.		$ ct.		$ ct.
J receives	334,02;	*L* do.	263,92;	*M* do	202,06

334,02+263,92+202,06=$800,00 Proof.

3. A copartnership of three persons had gained $900. *S* deposited $550 for 44 weeks; *T* $760 for 50 weeks; and *V* $840 for 40 weeks. What was the dividend for each?

Ans. *S*'s div. $227 34 8; *T*'s do. 356 99 4; *V*'s do. 315 65 8= $900 0 0 Proof.

4. Two merchants enter into partnership for 18 months. *A* at first puts in $1000; at the expiration of 6 months $400 more. *B* at first deposited $1600, and four months after took out $300. With this stock they lost $1200. What was the loss of each?

Ans. *A*'s loss $593,492; *B*'s do. 606,508=$1200,000 Proof.

5. *D* and *E* formed a connection in business for one year: *D* at the commencement advanced $1200; but *E* could not advance his share of funds until four months after. What must he then put in to have an equal share with *D*?

As 8*m.* : 12*m.* : : $1200 : *Ans.* $1800.

6. Three persons hired a coach to go out 50 miles and return, for 30*dol.* with the privilege of taking in more, if they pleased. Having gone 20 miles, they found an acquaintance desirous to accompany them the residue of the journey: 50 miles out they found two more, who wished to return with them. Each person is required to pay in proportion for the distance he rode. What is each one's proportional share of the carriage hire?

		$ ct.	$ ct.
Ans.	The 3 pay, each - - -	6 25	=18 75
	The 4th pays - - - -	5 00	= 5 00
	The 2 do. each - - -	3 12½	= 6,25
		Proof.	$30,00

Questions relative to Fellowship.

1. What is Fellowship? and on what principle is it founded?
2. How many kinds of Fellowship are there?
3. What is Single Fellowship?
4. What is the rule for Single Fellowship?
5. What is the proof?
6. What is Double or Compound Fellowship?
7. What is the rule?
8. What is the proof?

ANNUITIES.

An Annuity is a sum of money, payable at equal periods of time, either yearly, half, or quarter of a year, or for a certain number of years, or for ever.

When the debtor keeps the annuity in his own hands beyond the time of payment, it is said to be in *arrears*.

The sum of all the annuities for the time they have been forborne, together with the interest due on each, is called the *amount*.

If an annuity is bought off, or paid all at once at the beginning of the first year, the price paid for it is called the *present worth*.

I. To find the amount of an annuity at Simple Interest.

Rule.

1. Find the interest of the given annuity for 1 year.

2. Then 2, 3, &c. years, up to the given time, less one.

3. Multiply the annuity by the number of years given, and add the product to the whole interest, and the sum will be the amount sought.

Examples.

1. If an annuity of £100 be forborne 5 years; what will be

due for the principal and interest at the end of the said term, simple interest being computed at 6 per cent. per annum?

First year's interest of £100, at 6 per cent.					6 0
Second	do.	do.	do.	do.	12 0
Third	do.	do.	do.	do.	18 0
Fourth	do.	do.	do.	do.	24 0
Five years' annuity, at £100 a year - -					500 0
					£560 0 Amt.

Or thus:—Find the sum of the natural series of numbers 1, 2, 3, 4, &c. to the number of years less one. Multiply this sum by one year's interest, and the product will be the whole interest due upon the annuity. To this product add the product of the annuity and time, and the sum will be the amount sought

[*Take for instance the above example.*]

y. y. y. y. sum. £
1+2+3+4=10 100 One year, at 6 per cent. is £6
10×6=£60 £100×5*yr.*=500+60=£560 Amount requir.

2. If a house be let upon a lease of 6 years, at 200*dol.* per annum, what will be the amount for that time, at 7 per cent.?

sum. *dol.* *dol.* *dol.*
1+2+3+4+5=15 200 at 7 per cent.=14×15=210
$200×6= 1200

$1410 amt.

3. A house being let upon a lease of 7 years, at 300*dol.* per annum, and the rent being in arrear for the whole period; what is the amount due at the expiration of the time, simple interest being allowed at 6 per cent.? *Ans.* 2226*dol.*

The method of computing interest on notes and obligations, on which there are partial payments, very commonly adopted, is the following:—

RULE.

1. Find the amount of the principal for the whole time.
2. Compute the interest on the several payments, from the

time they were paid to the time of settlement, and find their amount.

3. Deduct the amount of the several payments from the amount of the principal.

Examples.

1. A note is given for $786, April 1st, 1828, at 6 per cent. interest, on which are the following endorsements, viz. :—

			$
August 1st, 1828, received		- - - -	179
December 1st, 1828,	do.	- - - -	105
April 1st, 1829,	do.	- - - -	200
March 1st, 1830,	do.	- - - -	350

How much remains due on the note, January 1st, 1831?

$		yr.	mo.
179,00	1st payment, August 1st, 1828.		
25,955	interest to January 1st, 1831 - - -	2	5
204,955	amount of 1st payment.		
105,00	2d payment, December 1st, 1828.		
13,125	interest to January 1st, 1831 - - -	2	1
118,125	amount of 2d payment.		
200,00	3d payment, April 1st, 1829.		
21,00	interest to January 1st, 1831 - - -	1	9
221,00	amount of 3d payment.		
350,00	4th payment, March 1st, 1830.		
17,50	interest to January 1st, 1831 - - -	0	10
367,50	amount of 4th payment.		
221,00	do. 3d do.		
118,125	do. 2d do.		
204,955	do. 1st do.		
$911,580	amount of payments.		

$ ct.		$ ct.	
786,00	principal.	915,69	amount of prin.
129,69	interest.	911,58	amount of pay.
915,69	amount of prin.	$ 4,11	remains due Jan. 1st, 1831.

2. A note is given for $1225, June 16th, 1827, at 6 per cent. interest; with the following endorsements. What will remain due August 1st, 1830?

		$	
Received	December 1st, 1827 - - -	250	Ans. $24,42,5.
Do.	July 16th, 1828 - - - - -	345	
Do.	January 1st, 1829 - - - -	500	
Do.	February 16th, 1830 - - -	200	

The following rule, established by courts of law in Massachusetts, for computing interest on notes, &c. on which endorsements are made, is practised not only in that State, but also in the State of New-York.

Rule.

"Compute the interest on the principal sum, from the time the interest commenced to the first time when a payment was made, which exceeds either alone, or in conjunction with the preceding payment, (if any) the interest at that time due; add that interest to the principal, and from the sum subtract the payment made at that time, together with the preceding payment, (if any) and the remainder forms a new principal; on which compute and subtract the payments as upon the first principal, and proceed in this manner to the time of final settlement."

A note of $850, dated May 1st, 1825, at 6 per cent. interest, has the following endorsements:—

	$
First endorsement, June 1st, 1826, for - - - -	150
Second endorsement, September 15th, 1826	12
Third endorsement, January 1st, 1827, for -	215
Fourth endorsement, July 16th, 1827, for - -	10
Fifth endorsement, November 1st, 1827, for	125
Sixth endorsement, May 1st, 1828, for - - -	350

How much remains due on the 1st of September, 1828?

Principal, May 1st, 1825 - - - - - -	850,00
Interest to June 1st, 1826, 13 months	55,25
Amount	$905,25
Paid June 1st, 1826 - - - - - - - - -	150,00

	$ cts.
Remainder for a new principal - - -	755,25
Interest to January 1st, 1827, 7 months	26,4337
Amount	$781,6837
Paid September 15th, 1826, (a sum less than the interest then due $ 12	
Paid January 1st, 1827 - - - - 215	
227	227,00
Remainder due for new principal - -	554,6837
Interest to November 1st, 1827, 10*m.*	27,7341
Amount	$582,4178
Paid July 16th, 1827 - - - - - $ 10	
Paid November 1st, 1827 - - - 125	
135	135,00
Remainder for new principal - - - -	447,4178
Interest to May 1st, 1828, 6 months -	13,4225
Amount	$460,8403
Paid May 1st, 1828 - - - - - - - - - -	350,00
Balance, due on note, May 1st, 1828,	$110,8403

Another method of computing interest, adopted by the superior court of the State of Connecticut, on notes on which there have been partial payments, is the following :—

RULE.

"Compute the interest to the time of the first payment, if that be one year or more from the time the interest commenced; add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner from one payment to another, till the payments are absorbed, provided the time between one payment and another be one year or more. But if any payment be made before one year's interest has accrued, then compute the interest on the principal sum due on the ob-

ligation for one year, add it to the principal, and compute the interest on the sum paid up to the end of the year; add it to the said sum paid, and deduct that sum from the principal and interest added as above, if the year does not extend beyond the time of final settlement: but if it does, then find the amount of the principal sum due on the obligation, up to the time of settlement; and likewise find the amount of the sum paid, from the time it was paid up to the time of final settlement, and deduct this amount from the amount of the principal. But if there be several payments made within the said time, find the amount of the several payments from the time they were paid to the time of settlement, and deduct their amount from the amount of the principal.

If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period."

Kirby's Reports, page 39.

Example.

1. A note of $1200, dated March 16th, 1825, at 6 per cent. interest, has the following endorsements, viz.:—

First payment, April 16th, 1826 - - - - - -	300
Second payment, January 1st, 1827 - - - - -	250
Third payment, May 1st, 1827 - - - - - - -	250
Fourth payment, June 15th, 1828 - - - - - -	250
Fifth payment, December 15th, 1828 - - -	5
Sixth payment, February 1st, 1829 - - - - -	6
Seventh payment, June 15th, 1829 - - - - -	250

What remains due on the note, July 1st, 1829?

$	
1200,00	principal, March 16th, 1825.
78,00	interest to April 16th, 1826, 13 months.
1278,00	amount.
300,00	deduct first payment.
978,00	balance due April 16th, 1826.
61,125	interest to May 1st, 1827, 12½ months.
1039,125	amount.

		$ ct.
	Second payment, Jan. 16th, 1827 - -	250,00
	Interest on 2d payment, 3½ months -	4,375
	Third payment, May 1st, 1827 - - -	250,00
504,375	Deduct 2d and 3d pay. and interest.	504,375
534,750	balance due, May 1st, 1827.	
36,095	interest to June 15th, 1828, 13½ months.	
570,845	amount.	
250,00	deduct 4th payment.	
320,845	balance due, June 15th, 1828.	
20,052	interest to July 1st, 1829, 12½ months.	
340,897	amount.	
	Fifth payment, December 15th, 1828, (less than the interest) - - - $	5,00
	Sixth payment, February 1st, 1829, (less than the interest) - - - - - -	6,00
	Seventh payment, June 15th, 1829 -	250,00
	Interest on 7th payment up to July 1st, ½ month - - - - - - - - - - - -	625
		261,625
261,625	deduct 5th, 6th, & 7th pay. & interest.	
$ 79,272	balance due on note July 1st, 1829.	

2. If a salary of £140, payable every half year, remain unpaid 7 years; what would it amount to at that time, at 6 per cent. per annum?

Note.—When the annuities are to be paid half yearly, then take half of the rate per cent., half of the annuity, and twice the number of years; and find the sum of the natural series from 1, 2, 3, &c. up to the whole number of terms or payments, less one: then multiply half the rate per cent. by half the sum of the natural series thus found: to this add the product of half of the annuity into twice the number of years; or what is the same, the whole of the annuity into the whole number of years.

If the annuity is to be paid quarterly, take quarter of the rate, quarter of the annuity, and four times the number of years. Find the natural series up to the number of terms less one: then

multiply quarter of the rate by quarter of the natural series, and to that product add quarter of the annuity multiplied into four times the number of years; or the whole annuity by the whole number of years, and the answer is obtained. Thus, in the last example, 4th.:—

The interest of £140, at 6 per cent. per annum, is £8 8s; ½ 84s. Twice the number of years is 14. Natural series of 13=91÷2=45½×84s.=£191 2s.

$$\overset{\pounds}{140}\times\overset{y.}{7}=\overset{\pounds}{980}: \text{ or } \overset{\pounds}{70}\times\overset{y.}{14}=\overset{\pounds}{980}: \text{ or } \overset{\pounds}{35}\times\overset{y.}{28}=\overset{\pounds}{980}$$

£191 2+980 0=£1171 2 amount.

Were the last annuity to have been paid quarterly, it would be found thus:

The interest of £140 is equal to £8 8s.÷4=£2 2s. or 42s. 4×7=28. Take the sum of the natural series to 27 inclusive =378÷4=94½×42s.=7y.×140l.: or 35l.×28y.=980l.=

£198 9s.+980 0=£1178 9s. *Ans.*

II. To find the present worth of an annuity at simple interest.

RULE.

Find the present worth of each year by itself, discounting from the time it falls due; and the sum of all these present worths will be the present worth required.

Examples.

1. What is the present worth of $500 per annum, to continue 3 years, at 6 per cent.?

dol.		*dol.*		*dol.*		*dol.*	
As 106						471,698=	Present worth of 1st yr.
112	:	100	: :	500	:	446,428=	do. 2d yr.
118						423,728=	do. 3d yr.
						1341,854=	*Ans:* $1341 85*ct.* 4*m.*

2. What is the present worth of 200*dol.* per annum, to continue 4 years, rebate being made at 7 per cent.?

Ans. 683*dol.* 89*ct.* 2*m.*

3. How much present money is equivalent to an annuity of 600*dol.* per annum, to continue 3 years, at a rebate of 6 per cent.? *Ans.* 1610*dol.* 22*ct.* 5*m.*

CONJOINED PROPORTION.

THIS rule is used, when the coins, weights, or measures of several countries are compared in the same question; or it is joining many proportions together, and by the relation which the several antecedents have to their consequents, the proportion between the first antecedent and the last consequent is discovered, as well as the proportion between the others in their several respects.

This rule may often times be abridged, by cancelling equal quantities which happen to be the same in both columns; and it is susceptible of proof, by as many statings in the Rule of Three, as the nature of the question may require.

CASE I.

When it is required to find how many of the first sort, either of coin, weight, or measure, mentioned in the question, are equal to a given quantity of the last.

RULE.

Place the numbers alternately, beginning at the left-hand, and let the last number stand on the left-hand column: then multiply the left-hand column continually together for a dividend, and the right-hand for a divisor, and the quotient will be the answer.

Examples.

1. Suppose 50 yards of America=50 yards of England; and 50 yards of England=25 canes of Toulouse; and 50 of Toulouse=80 ells of Geneva; and 80 of Geneva=160 ells of Hamburg: how many yards of America are equal to 512 ells of Hamburg?

50 of America = 50 of England.
50 of England = 25 of Toulouse.
50 of Toulouse = 80 of Geneva.
80 of Geneva = 160 of Hamburg.
512 of Hamburg =

$50\times25\times80\times160=16|000000$ divisor.

$50\times50\times50\times80\times512=5120|000000\div16=320yd.$ *Ans.*

$1\times16=16$ divisor.

Or by cancelling; $2\times5\times512=5120$ dividend.

$5120\div16=320yd.$ of America=512 ells of Hamburg.

[illegible]

8gr. Hence the standard weight for gold and silver coins, are in the ratio of about 1 to 16.

Names of Foreign Gold Coins.

	pwt.	gr.	$	ct.	m.
Johannes	18	0	16	0	0
Half Johannes	9	0	8	0	0
Doubloon	16	21	16	0	0*
Moidore	6	18	6	0	0
English Guinea	5	6	4	66	0
French Guinea	5	5	4	60	0
Spanish Pistole	4	6	3	60	0
French Pistole	4	4	3	65	0
English Sovereign	5	0	4	44	4
Half Sovereign	2	12	2	22	2

Names of Foreign Silver Coins.

	pwt.	gr.	$	ct.	m.
English or French Crown	18	17	1	10	0
Dollars of Spain, Sweden, and Denmark	17	8	1	0	0
English Shilling	3	18		22	2
Pistareen	3	11		20	0

The weights of the coins of the United States, as established by act of Congress, are as follows:—

	pwt.	gr.	$	ct.	m.
Eagle	11	6	10	0	0
Half Eagle	5	15	5	0	0
Quarter Eagle	2	$19\frac{1}{2}$	2	50	0
Dollar	17	8	1	0	0
Half Dollar	8	16		50	0
Quarter Dollar	4	8		25	0
Dimes	1	$17\frac{3}{4}$		10	0
Half Dimes		$20\frac{4}{5}$		5	0
Cents	8	16		1	0
Half Cents	4	8			5

The standard for pure gold coin is, 11 parts pure gold, and 1 part alloy; the alloy to consist of silver and copper. The standard for silver coin is, 1485 parts fine to 179 parts alloy; the alloy to be wholly of copper. Every cent to contain 208 grains of copper: and every half cent 104 grains.

* Usually 16 dollars.

From the preceding data of our own, and of many foreign coins and currencies, these various coins may be readily transposed from the one into another currency, by the rules of multiplication and division.

Questions relative to Exchange.

1. What is Exchange?
2. What is money?
3. How many kinds of money are there?
4. What is *real money?*
5. What is *imaginary money?*
6. What is meant by *par?*
7. What is the standard coin on the American Continent?
8. What is the standard coin in England and Ireland?
9. What is *course of exchange?*
10. What is *agio?*
11. What is the value of a pound sterling in federal money? and so of other foreign coins?
12. What is the value of a johannes, &c. &c.?
13. What is the value of the several coins in the United States?

Domestic exchange has been already fully explained in pages 145 to 148.

FOREIGN EXCHANGE.

Accounts are kept in England, Ireland, and most of the West India Islands, in pounds, shillings, pence, and farthings; although the value of their denominations are different in these several places. The readiest method of changing these, and all other currencies of the denominations of pounds, shillings, and pence, is by the rule given in pages 145 to 148.

In all other denominations, the answers in exchange are found, either by the Single Rule of Three, or by Practice.

Although the standard weights of most of the gold and silver coins have been given, as they have been rated by our government; yet their nominal value often fluctuates, which renders it necessary to ascertain the agio correctly.

HAMBURGH.

Accounts are kept in Hamburgh in marks, stivers, or shilling lubs. and derniers or grotes.

12 Derniers, or 2 grotes	- - -	1 Stiver, or shilling lubs.
16 Stivers, shilling lubs or 32 groats		1 Mark.
12 Grotes, or pence Flemish	-	1 Shilling Flemish.
20 Shillings Flemish	- - - -	1 Pound.

Note.

3 Marks	- - -	1 Rix dollar.
$7\frac{1}{2}$ Marks	- - -	1 Pound Flemish.=$2 50.

The *agio*, or rate of Hamburgh currency, is variable.

The mark banco is $33\frac{1}{3}$ cents, (by act of Congress.)

To change marks, &c. to Federal Money;—divide the marks by one third, as 3 marks make a dollar. If there are stivers, &c. annex them to the marks decimally, before the division by 3.

To change Federal Money into marks, &c.; multiply the marks, &c. by 3. These rules will apply in *all cases*, in which the value of the given currency is known in Federal Money.

Examples.

1. Change 5648 marks and 8 stivers to Federal Money.

$33\frac{1}{3}$*ct.*=$3)5648,5—for 8 stiv. 8 being the half of 16, viz. 1*m.*

$1882,833+

$ *ct. m.*
Ans. 1882,833
3

Proof 5648,500

Ans. 5648*m.* 8*st*

To change Hamburgh money to American, in pounds, shillings, pence, &c.

Rule.—As the given rate is to one pound, so is the Hamburgh sum to the American.

1. Change 1446 marks, 12 stivers, 1 grote, into New-England currency; exchange at 26*s.* 8*d.* Flemish to 1 pound New-England.

[illegible] icer [illegible]

[illegible] ency [illegible]

[illegible] d to [illegible]

[illegible] nued.

Change £250 New-England to Hamburgh money; exchange at 33s. 4d. Flemish to £1 New-England.

	£		s.	d.		£	
As	1	:	33	4	: :	250	
			12			400	
			400			2)100000	grotes.
						16)50000	stivers.
						3125	marks. *Ans.*

To reduce current to bank money.

Rule.—As 100 marks with the *agio* added is to 100 bank; so is the current money to the bank required.

Change 660 marks, 8 stivers, current, to bank; agio at 20 per cent.

cur. bank. cur.
As 120 : 100 : : 660,5 : $660,5 \times 100 \div 120 = 550,416$

Ans. 550*m.* 6*st.* 1*gr.*

To change bank to current money.

Rule.—As 100 marks is to 100 with agio added, so is the given bank to the current required.

Change 475 marks banco, to current; agio at 18 per cent.

m. cur. m.
As 100 : 118 : : 475 : =560*m.* 8*st.*

Or thus: $475 \times 18 = 85|50$ 475 bank.

16	85,8	
8|00	560.8	*Ans.* 560*m.* 8*st.*

Note.—The same rules will apply in all *other currencies*, to find the difference between bank and current money.

HOLLAND.

Accounts are kept in florins or gilders, stivers, grotes and pennings.

8	Pennings	- - - - - -	1	Grote.
2	Grotes	- - - - - - -	1	Stiver.
6	Stivers	- - - - - - -	1	Shilling.
20	Stivers	- - - - - - -	1	Florin or Gilder.
$2\frac{1}{2}$	Florins	- - - - - - -	1	Rix Dollar.
6	Florins	- - - - - - -	1	Pound Flemish.
5	Gilders	- - - - - -	1	Ducat.

The florin or gilder of Netherland is estimated in the United States at 40 cents; and a stiver at 2 cents.

To change florins into federal money, multiply them by 40 cents: and to change federal money into florins, divide the cents by 40.

Change 2540 gilders, 12 stivers, to federal money, at 40 cents per gilder.

```
   2540
    40ct.
 -------
 1016,24
 -------
     12
      2  cents per st.
     --
     24
     --
   Ans. $1016,24.
```

Or thus:

```
 gil.  st.
 2540  12
   20       sti. 1 gilder.
 ----
 50812  stivers.
     2  cents per sti.
 --------
 $1016,24 Ans.
```

Change 101624 cents into gilders, at 40 cents per gilder.

```
40)101624(2540,12
   ------
       24
       20 stivers.
      ---
  40)480(12
     480
     ---
  Ans. 2540gil. 12sti.
```

Or thus:

```
      ct.
  2)101624
    ------
 20)50812
    -----
   2540,12
  Ans. 2540gil. 12st.
```

To change Holland into New-England currency.

Rule.—As the given rate is to £1 New-England; so is the given sum to the New-England required.

Change 2122 gilders, 8 stivers, to New-England currency; exchange at 28*s.* 4*d.* to £1 New-England.

	s. d.		£		gil.	st.	
As	28 4	:	1	: :	2122	8	
	12*g.*					20	stivers 1 gilder.
	340				42448		stivers.
						2	grotes 1 stiver.

340)84896(249 13 10 2

Ans. £249 13*s.* 10*d.* 2*qr.* New-England.

What will 1650*lb.* of coffee amount to, at 15 stivers per *lb.*?

lb.
1650
15
2|0)2475|0 stivers.
1237=10

gil. *st.*
Ans. 1237 10

1237,5
40
$495,00.0 in fed. mo.

24750×2=$495,00 *Ans.*

lb. *ct.* *st.*
Or thus:—1650×30=15 *Ans.* $495,00.0

DENMARK.

Accounts are kept in Denmark in dollars, marks, and skillings.

16 Skillings - - - - - - - - -	1 Mark.
6 Marks, or 96 skillings, - - -	1 Rix dollar.
32 Rustics - - - - - - - - -	1 Copper dollar.
6 Copper dollars - - - - - -	1 Rix dollar.

A rix dollar of Denmark, is estimated at 100 cents. (United States' laws.)

How much will 9 pieces of platillas amount to, at 8 dollars. 44 skillings per piece?

dol. *ski.*
8 44
9
76 12

44
9
96)396(4
384
12

76 Rix dollars, 12 skillings.

BREMEN.

Accounts are generally kept in rix dollars and grotes: 72 grotes make a rix dollar, which is equal to 2¼ marks.

ANTWERP.

Accounts are kept in Antwerp in gilders, shillings, and grotes.

12	Grotes - - - - - -	1 Shilling.
3⅓	Shillings, or 40 grotes,	1 Gilder.

A grote in Antwerp is equal to 1 cent of the United States; a gilder being reckoned at 40 cents; and stivers are of the value of 2 cents of the United States.

What sum must be paid in New-York for an invoice of goods imported from Antwerp, amounting to 6370 gilders, exchange 40 cents per gilder, at an advance of 50 per cent.?

gil.	6370	
6370	3185	advance.
50 per ct. ad.		
	9555	
3185|00	40	cents.
	3822,00	

Ans. $3822.

RUSSIA.

Accounts are kept in Petersburgh, more commonly, in rubles and copees, reckoning 100 copees to 1 ruble. A ruble, in 1802, was estimated in the United States at 55 cents. The agio varies.—The following have been their currencies, viz.:—

18 Pennings	- - - - - -	1 Gross.
30 Gross	- - - - - - - -	1 Florin.
3 Florins	- - - - - - -	1 Rix dollar.
2 Rix dollars	- - - - -	1 Gold ducat.

How many rubles must be received in Petersbugh for a bill of 12500 gilders on Amsterdam, when the exchange is 30 stivers per ruble?

	st.		cop.		gil.	
As	30	:	100	: :	12500	:

20

250000 stivers.
100

3|0)2500000|0

8333,33⅓

Ans. 8333*rub.* 33⅓*cop.*

A bill of £5000 is drawn on New-York, exchange at 55 cents per ruble. What is its value in Petersburgh?

	ct.		ru.		$	£	$
As	55	:	1	: :	12500	5000,0÷4=12500.	

,55)12500,00(22725,27 *Ans.* 22725*rub.* 27*cop.*

FRANCE.

12 Deniers	- - - - - - - - -	1 Sol.
20 Sols	. - - - - - - - -	1 Livre.
3 Livres	- - - - - - - - -	1 Crown.

A livre of France is valued at 18½ cents in the United States; consequently a sol is less than a cent; and a denier less than a mill.

Change 5436 livres into federal money, exchange at 18½ cents per livre.

5436×18½*ct.*=1005,66*ct.* *Ans.* $1005,66*ct.*

Change the last answer into livres.

ct.m. $ *ct.*
18,5)1005,660(5436 Livres.

Money of account in France is generally reckoned in francs, decimes, and centimes. A franc is valued at 18 cents 7,3⅛ mills; or 18⅓ by some: 10 centimes=1 decime: 100 centimes or 10 decimes=1 franc. To change francs to livres, multiply by 81, and divide by 80: and to change livres to francs, multiply by 80, and divide by 81. The reason of this is, that the value of a livre, viz. 18 cents 5 mills, is proportioned to the value of a franc, viz. 18 cents 7 mills, 3125, as 80 to 81.

To reduce sols and deniers to centimes or hundredths, take half of the sols and deniers, as if integers, and this half is the number of the centimes required sufficiently near.

SPAIN.

4	Marvadies	1	Quarto.
34	Marvadies	1	Real vellon.
34	Marvadies of real vellon	1	Real of plate.
8	Reals of plate	1	Piastre.
10	Reals of plate	1	Dollar.
5	Piastres	1	Spanish pistole.

The real plate is estimated at 10 cents; and the real vellon at 5 cents in the United States. As a real plate is 10 cents or 1 dime; it is reduced to federal money by calling the reals so many dimes; or cents by annexing a cipher to the reals: and reals vellon are changed into federal money by multiplying them by ,05.

As in Spain, reals plate are not the double of reals vellon, for 17 reals plate are equal to only 32 reals vellon; therefore to change reals vellon to reals plate, multiply by 17, and divide by 32; or to change reals plate to reals vellon, multiply by 32, and divide by 17.

Change 1600 reals vellon to reals plate.

$\overset{r.v.}{1600}\times17\div32=\overset{r.p.}{850}$ And $\overset{r.p.}{850}\times32\div17=1600$ Reals vellon.

To change these into federal money:

$1600\times,05=\$80,00$ $\$85,0$ or $85,00$. *Ans.*

It is thus evident, that 100 reals plate$=188\frac{4}{17}$ reals vellon: for $100\times32\div17=188\frac{4}{17}$.

CADIZ.

8½	Quartos	1	Real vellon.
20	Real vellons	1	Dollar of plate.

To bring reals plate to dollars.

Rule.—Multiply the given sum by 32, and divide by 17 for reals vellon, then divide the reals vellon by 20 for dollars.

In 640 reals plate, how many dollars?

$640\times32\div17=1204\frac{12}{17}$ Reals vellon. $\overset{r.v.}{1204}\div20=\overset{\$}{60}\ \overset{r.}{4}$

$\frac{12}{17}$ $12\times8\frac{1}{2}q.\div17=6q.$ *Ans.* $\$60\ 4r.\ 6q.$

To change dollars into reals plate:

Multiply the dollars by 20 for reals vellon, and those by 17, and divide by 32, will give the reals plate required: or multiply the dollars by $10\frac{5}{8}$ for reals plate.

In 32 dollars, how many reals plate?

$32\times20\times17\div32=340$*r. p.*: or $32\times10+325\div8=340$*r. p.*

BILBOA.

Accounts are kept in reals vellon, and maravadies; 34 maravadies making 1 real.

BARCELONA.

Accounts are kept in Barcelona and the whole province of Catalonia, in livres, sols, and deniers.

12 Deniers	- - -	1 Sol.
20 Sols	- - - -	1 Livre.
$37\frac{1}{2}$ Sols, or $1\frac{7}{8}$ livres	- -	1 Hard dollar.
28 Sols	- - - -	1 Current dollar, the pias. of ex.

To change livres to hard dollars:—divide the livres by 3 and then by 5, and add the two quotients together for hard dollars.

How many hard dollars in 630 livres?

$630\div3=210\div5=42$ $210+42=252$ Hard dollars. *Ans.*

To change hard dollars to livres:—add to the given sum, the half, quarter, and eighth of it, and the sum will be the livres required: or double the livres, and deduct one sixteenth.

In 384 hard dollars, how many livres?

```
2|384          384          768
2|192     or     2           48
2| 96          ———          ———
 | 48       16)768(48       720 livres. Ans.
  ———          64
720 livres.    ———
               128
               128
               ———
```

To change livres to current dollars: multiply the livres by 5, and divide that product by 7 for current dollars.

To change current dollars to livres, multiply the current dollars by 7, and divide that product by 5 for livres.

Change 1358 livres to current dollars.

$1358 \times 5 = 6790 \div 7 = 970$ current dollars. *Ans.*

Change 970 current dollars to livres.

$970 \times 7 = 6790 \div 5 = 1358$ livres. *Ans.*

PORTUGAL.

Accounts are kept in reas and millreas.

1000 Reas make 1 Millrea ;=$1 24*ct.* federal money ; or 5*s.* 7½*d.* sterling.

To reduce millreas into federal money, multiply by 125, and the product will be cents and decimal parts of a cent.

Note.—When the reas are less than 100, place a cipher before them. Thus,

Example.—320,48 will be $320{,}048 \times 125 = 400{,}05750$.

Ans. $400 05*ct.* 7½*m.*

To reduce cents to millreas, divide the cents by 125 : if there are decimals, carry on the quotient to 3 places of decimals ; the whole numbers will be millreas, and the decimals reas.

In 5280 cents, how many millreas?

5280*ct.*÷125*ct.*=42,320 *Ans.* 42*m.* 320*r.*

LEGHORN.

Accounts are kept in piastres, soldi, and denari ; reckoning 12 deniers to 1 soldi, and 20 soldi to 1 piastre or dollar, valued at 48*d.* sterling at par. One pezza of 8 reals, is valued at about 86½ cents.

NAPLES.

Accounts are kept in ducats and grains, reckoning 100 grains to 1 ducat. The current coins are grains, carlins, ducats, dollars, and ounces.

The Naples dollar passes for 120 grains ; and the Spanish dollan for 126 grains. One ducat of Naples=75½ cents.

TRIESTE.

Accounts are kept in florins and kreutzers: 60 kreutzers make 1 florin. The exchange on London has been 12 florins for a pound sterling:—the other kinds of moneys are, soldi and livres.

20 Soldi - - - - - - - - -	1 Livre.
5¼ Livres - - - - - - - - -	1 Florin.

GENOA.

Accounts are kept in denarii, soldi, and pezzos or lires.

12 Denarii - - -	1	Soldi.
20 Soldi - - - -	1	Pezzo or lire,=85*ct.* (nearly.)
1 Pezzo of exchange	5¾	Lires.

In Milan,	1 Crown=	80	Soldi of Genoa.
" Naples,	1 Ducat =	86	do.
" Leghorn,	1 Piastre=	20	do.
" Sicily,	1 Crown=	127⅝	do.

VENICE.

Venice has three kinds of money: viz.—banco money, current money, and picoli money. Banco money is 20 per cent. above banco current; and banco current 20 per cent. above picoli. The different denominations are denarii, soldi grosi, and ducats.

12 Denarii - - - - - -	1 Soldi, or sol d'or.
5⅙ Soldi - - - - - - - -	1 Gros, or grosi.
24 Gros, or grosi - - -	1 Ducat.

100 Ducats banco of Venice in	Leghorn	= 93	Pezzos.
— — —	Rome	= 68½	Crowns.
— — —	Lucca	= 77	do.
— — —	Frankfort	=139½	Florins.

The par of exchange in London has been 50¼*d.* sterling per ducat.

WEST INDIES.

In *Jamaica* and *Bermudas*, a Spanish dollar passes for 6*s.* 8*d.*; three dollars, 20*s.* or one pound, Jamaica currency. They compute in pounds, shillings, and pence.

BARBADOES.

In Barbadoes currency, a Spanish dollar is valued at 8*s.* 3*d.*

MARTINICO, TOBAGO, and ST. CHRISTOPHERS.

In these islands, inhabited by French and English, the former keep their accounts in livres, sols, and deniers; and the latter in pounds, shillings, and pence. A current dollar passes for 8*s.* 3*d.*; a round dollar for 9*s.*

FRENCH WEST INDIES.

Their currency is similar to France.

12 Deniers	- - - - - - - - - -	1 Sol.
20 Sols	- - - - - - - - - - -	1 Livre.

SPANISH WEST INDIES.

Accounts are kept in the Spanish West India Islands, in dollars and reals, computing 8 reals to a dollar.

EAST INDIES.—CALCUTTA.

Accounts are kept in rupees, annas, and pice.

12 Pice make 1 Annas; 16 Annas 1 Rupee.

BOMBAY.

Accounts are kept in rupees, quarters, and rees.

100 Rees	- - - - - - -	1 Quarter.
4 Quarters	- - - - - - -	1 Rupee.

The current money is in mohurs, rupees, and pice.

50 Pice	- - - - - - - - - -	1 Rupee.
15 Rupees	- - - - - - -	1 Mohur.

MADRAS.

Accounts are kept in pagodas, fanams, and cash.

80 Cash	- - - - - - - - - -	1 Fanam.
36 Fanams	- - - - - - - - -	1 Pagoda.

A pagoda, by act of Congress, is valued at 184 cents.
The Bengal, or sicca (new) rupee, is valued at 50 cents.

BATAVIA.

Accounts are kept in rix dollars and stivers.

The rix dollar is 48 stivers; the ducatoon 80 stivers.
The Spanish dollar is from 60 to 64 stivers.

CHINA.

Calculations are made in tales, mace, candareens, and cash.

10 Cash	- - -	1 Candareen.
10 Candareens	- -	1 Mace.
10 Mace	- - -	1 Tale.

A tale of China, by the United States' laws, is estimated at 148 cents. A Spanish dollar is current at 72 candareens.

Their weights are in tales, piculs, and cattas.

16 Tales	- - - -	1 Catta.
100 Cattas	- - - -	1 Picul.

A picul is equal to $133\frac{1}{3}$*lb.* English.

To change English or American *lb.* to cattas:

Rule.—Deduct one quarter from the English or American for cattas.

To change cattas to American or English:

Rule.—Add one third of the cattas for the American or English.

In 31344*lb.* American, how many cattas?

$$\begin{array}{r} 4|31344 \\ 7836 \\ \hline \end{array}$$

Ans. 23508 Cattas.

In 23508 cattas, how many *lb.* American?

$$\begin{array}{r} 3|23508 \\ 7836 \\ \hline \end{array}$$

Ans. 31344

How many dollars will pay for an invoice of tea, amounting to 3566 tales, 1 mace, 6 candareens?

t. m. c.
Spanish dollar=72)3566 1 6(4953 *Ans.* $4953

MANILLA.

Computations are made in dollars, reals, and quartos.

12 Quartos	- - - -	1 Real.
8 Reals -	- - - -	1 Dollar.

Their 100*lb.* are equal to 104*lb.* American.

Their weights are, 25*lb.* make an arobe; $5\frac{1}{2}$ arobes 1 picul.

COLUMBO, ISLAND OF CEYLON.

Their money is in paper, silver, and gold.

Paper money is in the bills of the company, and is of uncertain value.

Silver is in rupees of the different parts of India.

The sicca rupee is the most valuable by 7 to 8 per cent.

Gold is the mohur pagoda. The exchange is various, as silver is seldom seen.

6 Stivers	- - -	1 Shilling Flemish.
8 Shillings	- - -	1 Rix dollar.
30 Stivers	- - -	1 Rupee.
64½ Stivers	- - -	1 Spanish dollar.

JAPAN.

Their accounts are in tales, mace, and candareens.

10 Candareens	- - -	1 Mace.
10 Maces	- - - -	1 Tale=75ct. or $¾.
10 Maces are equal to	- -	1 Rix dollar.

6 Tales make a corban, a gold coin not used in accounts.

10 Tales make 1 mace; and 16 maces 1 catta in weight.

ALLIGATION.

Alligation respects the mixture of several simples of different qualities, so that the composition may be of a mean or middle quality. It consists of two kinds; viz. Alligation Medial; and Alligation Alternate.

ALLIGATION MEDIAL,

Is when the qualities and prices of several things are given to find the mean price of the mixture which is composed of these materials.

Rule.

As the sum of the several quantities is to the whole value of these quantities; so is any part of the composition to its mean

price: or, as the sum of the several quantities is to any part of the composition; so is the whole value to its value.

Proof.—The value of the whole mixture, at the mean price, must equal the whole value of the several quantities at their respective prices.

Examples.

1. If 10 gallons of wine, at 70 cents per gallon; 8 gallons, at 80 cents; 5 gallons, at 60 cents; and 6 gallons, at 125 cents, be mixed together; what is 1 gallon of this mixture worth?

gal.	*ct.*
10 at	70=7,00
8 -	80=6,40
5 -	60=3,00
6 -	125=7,50
29	23,90

gal. \$ *ct.* \$
As 29 : 23,90 : : 1
23,90÷29=82*ct.* 4*m.*+*Ans.*
gal. *gal.* \$ *ct.*
Or, as 29 : 1 : : 23,90 :
Ans. 82*ct.* 4*m.*+

Proof. 82*ct.* 4*m.*×29= *Ans.* \$23 90*ct.*

2. A farmer mixed 20 bushels of wheat, at 110 cents a bushel, with 16 bushels of rye, at 56 cents a bushel, and 12 bushels of corn, at 60 cents per bushel: what was 1 bushel of the mixture worth? *Ans.* 79*ct.* 5*m.* per bushel.

3. A grocer would mix several kinds of sugar together for a more speedy sale; viz. 70*lb.* at 8 cents; 80*lb.* at 10 cents; 60*lb.* at 11 cents; and 40*lb.* at 12½ cents: how must he estimate this mixture by the pound? *Ans.* 10*ct.* 1*m.* per pound.

4. A silversmith mixes 8*oz.* of silver, at 80 cents per ounce, with 12*oz.* at 70 cents per ounce: what is the value of 1*oz.* of the mixture? *Ans.* 74*ct.*

4. A tobacconist mixed 40*lb.* of tobacco, at 20 cents; 50*lb.* at 22 cents; and 60*lb.* at 25 cents per *lb.*: what is the price of 1*lb.* of the mixture? *Ans.* 22*ct.* 66*m.*+

5. If 90*gal.* New-England rum, at 34 cents per gallon; and 110*gal.* of whisky, at 28 cents per gallon, be mixed with 40*gal.* of water: what is the mixture worth by the gallon? *Ans.* 26*ct.* 41*m.*+per gallon.

6. A goldsmith mixes 10*lb.* 8½*oz.* of gold of 15 carats fine, with 12*lb.* 6½*oz.* of 19 carats fine: what is the fineness of the mixture? *Ans.* $17\frac{46}{185}$ carats.

7. A refiner melts $3\frac{1}{2}lb.$ of gold of 20 carats fine, with $5\frac{1}{2}lb.$ of 16 carats fine: how much alloy must he put in to make it 21 carats fine? *Ans.*—It is not fine enough by $2\frac{1}{8}$ carats: he must therefore add more gold.

8. A refiner melts again $2\frac{1}{2}lb.$ of gold of 22 carats fine, and $3\frac{1}{2}lb.$ of 20 carats fine: must he now add alloy or not, to make it 20 carats fine? *Ans.* $20\frac{5}{6}$ fine. It is too fine by $\frac{5}{6}$ of a carat.

ALLIGATION ALTERNATE,

Is the method of finding what quantity of each of the simples, whose rates are given, will compose a mixture of a given rate. Hence it is the reverse of Alligation Medial, and may be proved by it.

Case I.

When the mean rate of the whole mixture, and the rates of all the ingredients are given without any limited quantity.

Rule.

1. Write the rates of the simples under each other, and set the mean rate at the left-hand of them.

2. Link each rate which is less than the mean rate, with one or more that is greater.

3. Take the difference between each rate, and the mean price, and set it opposite to that rate with which it is linked.

4. If only one difference stand against either rate, it will be the quantity required at that rate; but if there be several, their sum will be the quantity.

Note.—If all the given prices be greater or less than the mean rate, they must be linked to a cipher. Different modes of linking will produce different answers.

Examples.

1. A grocer has several sorts of sugars, viz. at 8, 9, 11, and 12 cents per pound: how much of each sort must he take to make a mixture, worth 10 cents a pound?

	ct.	lb.		
10	8	2	of	8=16
	9	1	"	9= 9
	11	1	"	11=11
	12	2	"	12=24
		6		6)60(10
				60

10ct. Proof.

	ct.	lb.
10	8	2+1=3× 8=24
	9	2+1=3× 9=27
	11	2+1=3×11=33
	12	2+1=3×12=36
		12 120

120÷12=10ct.

The more they are linked, the greater the several quantities

2. A goldsmith would mix gold of 17, 19, 21, and 23 carats fine, so that the compound may be 20 carats fine: what quantity of each must be taken?

20	17	3×17 51
	19	1×19=19
	21	1×21=21
	23	3×23=69
		8 8)160(20ct.
		16
		0

20	17	3+1=4×17=68
	19	1+3=4×19=76
	21	1+3=4×21=84
	23	3+1=4×23=92
		16 320

320÷16=20ct. *Ans.*

3. A vintner would mix several sorts of wine, viz. at 8s. 10s. 11s. and 12s. per gallon, with water, that the mixture may be worth 9s. a gallon: how much of each sort must be taken?

9	0	3+2	5× 0= 0
	8	3+2+1	6× 8= 48
	10	1	1×10= 10
	11	9+1	10×11=110
	12	9+1	10×12=120
			32 32)288(9
			288

Proof 9s. *Ans.* 32 gallons; of which there are 10*gal.* at 12s.; 10*gal.* at 11s.; 1*gal.* at 10s.; 6*gal.* at 8s.; 5*gal.* of water.

4. A grocer had two sorts of tea; one at 7s. and one at 9s. per pound: how must he mix them to afford the composition at 8s. per pound?—*Ans.* he must mix equal quantities of each.

5. How much water can be mixed with wine worth 8s. 4d. per gallon, to afford the mixture at 5s. 3d.?

Ans. 63 gallons of wine and 37 of water.

6. A butcher had a few pieces of meat remaining in his stall, which he estimated at the following prices; viz. at 5*d.* 6*d.* 8*d.* and 10*d.* per *lb.*; he was offered 7*d.* a *lb.* and take all: how many pounds did he have?

Ans. 14*lb.* viz. 4*lb.* at 5*d.*; 4*lb.* at 6*d.*; 3*lb.* at 8*d.*; & 3*lb.* at 10*d.*

7. How many bushels of wheat, at 95*ct.* per bushel; rye at 62*ct.*; barley, at 70*ct.*; and corn at 64*ct.* may be sold, so that the average rate per bushel is 66 cents? *Ans.* 78*bus.* viz. 33 of rye; 33 of corn; 6 of barley; and 6 of wheat.

Case II.

ALLIGATION PARTIAL.

When one of the ingredients is limited to a certain quantity, to find the several quantities of the rest, in proportion to the quantity given.

Rule.

1. Take the difference between each price and the mean rate, and place them alternately, as in Case I.

2. Then, as the difference standing against the simple, whose quantity is given is to that quantity, so is each of the other differences severally, to the several quantities required.

Examples.

1. A merchant would mix 20*gal.* of wine, at 112*ct.* with some at 96, 84, and 60*ct.* per gallon, so that a gallon of the mixture may be sold at 90*ct.*: what quantity of each must be taken?

ct.

$$90\begin{cases}112 - 30 \\ 96 - 6 \\ 84 - 6 \\ 60 - 22\end{cases}$$

112—30 Stands against the given quantity.

Then, as

	gal.		gal.		ct.	2240
30 :	20 ::	6 :	= 4	at	96 =	384
30 :	20 ::	6 :	= 4	"	84 =	336
30 :	20 ::	22 :	$=14\frac{2}{3}$	"	60 =	880
			$42\frac{2}{3}$			3840

$3840 \div 42\frac{2}{3} = 90$*ct.* Proof.

Note.—Questions in this rule, and in Alligation generally, admit of a vast variety of answers, which are exclusively dependent on the manner in which they are linked. By linking

them differently from the above, other numbers, bearing the same proportion between themselves will be produced.

2. How many gallons of water may be mixed with 120*gal.* of brandy, worth 9*s.* per gallon, so that it may be offered at 8*s.* the gallon ?

$8\left\{\begin{array}{l} 9 \text{—} 8 \\ 0 \text{—} 1 \end{array}\right.$ difference against the given quantity.

As 8 : 120 : : 1 : = 15*gal.* water.

w.	*b.*	
15+120=135×8=1080	120×9=1080	Proof.

3. A merchant mixed with 57*gal.* of rum, valued at 8*s.* per gallon, other rum at 6*s.* 8*d.* per gallon, and some water, when he found it stood him in 6*s.* 4*d.* per gallon : how much rum and water did he add ?—*Ans.* 57*gal.* at 6*s.* 8*d.* ; and 18*gal.* of water.

4. How much gold of 15, 16, 20, and 24 carats fine, of which the 20 carats is of the fineness of 10*oz.* must be mixed, so that the mixture will be 18 carats fine ?

Ans. 52*oz.* viz. 16*oz.* of 15 ; 16*oz.* of 16 ; 10*oz.* of 20 ; and 10*oz.* of 24.

Case III.

When the whole composition is limited to a given quantity.

Rule.

1. Place the difference between the mean rate and the several prices alternately, as in Case I.

2. Then, as the sum of the differences thus determined is to the given quantity or whole composition, so is the difference of each rate to the required quantity of each rate.

Examples.

1. A grocer had sugars at 8, 9, 11, and 12 cents per pound; and he wished to make a composition of 224*lb.* worth 10*ct.* per pound : what quantity of each did he take ?

				lb.	*lb.*		*lb.*		*ct.*	
10	8	2+1=3	As	12 :	224 :	3 :	56	at	8	per *lb.*
	9	1+2=3				3 :	56	"	9	
	11	1+2=3				3 :	56	"	11	
	12	2+1=3				3 :	56	"	12	
		12					224			

2. How many gallons of water may be mixed with wine, worth 8*s*. 6*d*. a gallon, to fill a cask of 120 gallons, so as to be afforded at 7*s*. per gallon?

$$84\left\{\begin{array}{r|r} \textit{d.} & \\ 102 & 84 \\ 0 & 18 \\ \hline & 102 \end{array}\right. \qquad \text{As } 102 : 120 :: \left\{\begin{array}{l} 84 : 98\tfrac{14}{17} \text{ Wine.} \\ 18 : 21\tfrac{3}{17} \text{ Water.} \\ \hline \quad\;\; 120.0 \textit{ Ans.} \end{array}\right.$$

3. A grocer has currants at 6, 8, 11, and 12*ct*. per pound, of which he would make a mixture of 150*lb*. so as to afford them at 9*ct*. per pound: how much of each sort must he take?

Ans. $37\frac{1}{2}$*lb*. of each sort.

4. A silversmith has three sorts of silver bullion; one of 6; one of 9; and one of 10*oz*. fine; and he would mix 21*oz*. of it so as to be 8*oz*. fine: how much of each sort must he take?

Ans. 9*oz*. of the 6; and 6*oz*. each of the 9 and 10*oz*.

Questions relative to Alligation.

1. What is Alligation?
2. Of how many kinds does it consist? and what are they?
3. What is Alligation Medial?
4. What is the rule; and what the proof?
5. What is Alligation Alternate?
6. Is it similar, or the reverse of Alligation Medial?
7. When the mean rate of the whole mixture, and the rates of all the ingredients are given, without any limited quantity: what is the first step in the rule? what the second? what the third? and what the fourth?
8. When all the given prices are either greater or less than the mean rate, to what must they be linked?
9. Will different methods of linking produce different answers
10. What is Alligation Partial?
11. What is the first step in the rule? what is the second?
12. When the whole composition is limited to a given quantity, what is the first step in the rule? what is the second?

POSITION.

POSITION is a rule, which, by false or supposed numbers taken at pleasure, discovers the true numbers required. It is divided into two kinds, Single and Double.

SINGLE POSITION,

Is, when by using a supposed number, and performing the same operations with it as are described in the given question, the true number is obtained by the following

Rule.

As the sum of the errors, or the result of the supposed operation is to the given sum; so is the proposed number to the true number required.

Proof.—Work with the answer agreeable to the directions given in the question, and the result will equal the given number.

Examples.

1. Three merchants, *A*, *B*, and *C*, purchased a quantity of wine for \$390; of which *A* paid three times as much as *B*; and *B* three times as much as *C*: how much did each pay?

Suppose *C* paid	20	20	*C.*
	3	60	*B.*
	—	180	*A.*
Then *B* would pay	60	—	
	3	260	Sum of the errors.
	—		
And *A* pay - -	180		

As \$260 : \$390 :: 20 supposed number.

```
          20
         ———
260)7800(30 C's share.
     780
     ———
       0
```

30×3=90 *B's* share. 90×3=270 *A's* share.

270+90+30=\$390 *Ans.*

2. In compliance with invariable practice in the illustration of this rule, let the old schoolmaster be again called upon to inform us, what is his present number of pupils. If he answer, that if he had as many more as he now has, half as many, one third, and one fourth as many, his present number would equal 185: what would in reality be the present number of this old preceptor?

Suppose he had	36
As many more	36
Half as many	18
$\frac{1}{3}$ as many	12
$\frac{1}{4}$ as many	9
Sum of the errors	111

As 111 : 185 : : 36
36
———
1110
555
———
111)6660(60 true No.
666
———
0 *Ans.* 60.

$60+60+30+20+15=185$ Proof.

3. What number is that, which being increased by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{12}$, and $\frac{1}{15}$ of itself, will amount to 162? *Ans.* 60.

4. Three boys divide a prize of $210, so that *Sam* has half as much as *Peter*, and *David* four times as much as *Sam*: what were their several shares?

Ans. Peter had $60; *Sam* $30; and *David* 120=$210.

4. A person after spending $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{8}$ of his money, had $140 left: what had he at first?

Suppose he had	$240
$\frac{1}{3}$	80
$\frac{1}{4}$	60
$\frac{1}{8}$	30
	170
He had left	70

$ *lt.* *s. sum.* *left.*
As 70 : 240 : : 140
140
———
7,0)3360,0
———
480 *Ans.*

$480\div\frac{1}{3}=160$ $480\div\frac{1}{4}=120$ $480\div\frac{1}{8}=60$ 140 left.

$160+120+60+140=480$ Proof.

5. A certain sum is to be divided between 5 persons, so that the first shall have $\frac{1}{3}$, the second $\frac{1}{4}$, the third $\frac{1}{6}$, the fourth $\frac{1}{8}$, and the fifth the remainder, which is $18. what was the sum divided? *Ans.* 144.

6. A person lent his friend a sum of money, unknown, at 6 per cent. simple interest; at the end of 4 years he received the principal and interest, viz. $806: what was the sum lent?

Ans. $650.

DOUBLE POSITION.

Double Position teaches to resolve questions by making two suppositions of false numbers.

Rule.

1. Take any two convenient numbers, and proceed with each according to the conditions of the question.

2. Find how much the results are different from the results in the question.

3. Multiply the first position by the last error, and the last position by the first error.

4. If the errors are *alike*, divide the difference of the products by the difference of the errors, and the quotient will be the answer.

5. If the errors are *unlike*, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

Note.—The errors are said to be *alike*, when they are both too great or too small: and *unlike*, when one is larger or *plus*, and the other *minus* or less.

Examples.

1. A legacy of $1000 is to be divided among three persons, *A*, *B*, and *C*, so that *B* shall receive $50 more than *A*, and *C* $100 more than *B*: what is the share of each?

Suppose *A* received	200	Suppose *A* received	250
Then *B* received -	250	Then *B* - - - - - -	300
And *C* received - -	350	And *C* - - - - - - -	400
Amount of errors -	800		950
Subtracted from	1000	Given sum 1000—950=50	
Error less	200	Error less 50.	

sup.
200 X 200—
250   50—
200   200

50000   10000
10000

40000 differ. of prod.

Diff. of er. 15|0)4000|0(266,666
30

100
90

100
90

100
90

10

A's share $266,666
B's "   316,666
C's "   416,666

Proof 1000,000

2. Said *Richard* to *James*, what did your new suit of clothes cost you? *James* replied, if they had cost me 5 times as much, and $25 more, I should have paid $400: how much did they cost?

Suppose they cost $60
5

300
Add 25

Too little by 75. 325

Suppose $80
5

400
Add 25

Too much 25. 425

60 X 75—
80   25+
75   60

6000   1500

Errors unlike.

Sum of pro. { 6000
1500

Sum of Errors 1,00)75,00(75
Ans. $75×5=375+25=
400 Proof.

3. *A* and *B* have both an equal income. *A* saves one sixth of his yearly income; but *B*, by spending $200 a year more than *A*, at the end of 6 years finds himself $600 in debt: what is their annual income, and how much do they spend yearly?

Ans. Their income is $600 yearly; *A* spends $500; and *B* $700 annually. Hence *B* is $600 in debt.

4. There is said to be a fish, (probably the sea serpent) whose head is 10 feet long; his tail is as long as his head and half his body; and his body is as long as his head and tail: what is the whole length of this enormous fish?

	ft.
Suppose his body	20
His head - - - -	10
His tail - - - - -	20

Now his body must be as long as his head and tail; but it falls short by 10 feet.

	ft.
Suppose his body is	30
Head - - - - - - - -	10
Tail - - - - - - - - -	25
Body falls short - -	5

$$\begin{array}{rr} 20 & 10- \\ & \!\!\!\!\!\!\!\times \\ 30 & 5- \\ 10 & 20 \\ \hline 300 & 100 \\ 100 & \\ \hline \text{Diff. } 5)200 & \\ \hline 40 & \end{array}$$

	ft.
Body - - - -	40
Head - - - -	10
Tail - - - -	30
Ans.	80

5. A labourer was hired for 80 days, upon these conditions; that for every day he laboured, he should receive $1; and for every day he was idle, he should forfeit 30 cents. At the expiration of the time he received $41; how many days did he work, and how many was he idle?

Ans. He laboured 50 days, and was idle 30 days.

6. A gentleman had 2 valuable horses and a carriage, worth $400. When the first horse was harnessed in it, the carriage and horse together were twice the value of the second horse: but when both horses were harnessed in it, the horses and carriage were triple the value of the second horse: what was the value of each horse?—*Ans.* The 1st horse was $200; 2d $300.

7. Divide 24 into two such parts, that if the greater were multiplied by 3, and the less by 9, the products will be equal.

Ans. 18 and 6.

8. A drover, having disposed of his cattle in market, received for the whole $1200: viz. for an ox $50; for a cow $25; and for a calf $5. He had twice as many cows as oxen, and twice as many calves as cows: how many were there of each sort?

Ans. 10 oxen: 20 cows: and 40 calves.

9. An instructer being asked by his pupil, what is the clock; replied, that it was $\frac{7}{16}$ of the time from the last to the following midnight; and he would commend him if he would work out the answer. *Ans.* 10 *h.* A. M. or half-past 10 o'clock, A. M.

10. A gentleman and lady were married, whose ages were proportioned to each other, as 4 to 3. Twelve years after marriage, they found their ages were proportioned as 6 to 5: what were their respective ages when married?—*Ans.* 24 & 18.

Questions relative to Position.

1. What is Position?
2. Of how many kinds does it consist? and what are they called?
3. What is Single Position?
4. What is the rule? and what is the proof?
5. What is Double Position?
6. What is the first step in the rule? what the second? what the third? what the fourth? what the fifth?
7. How are the errors marked?
8. When are the errors said to be alike? and when unlike?

INVOLUTION;

OR THE RAISING OF POWERS.

A power is produced by multiplying any given number into itself continually a given number of times. If a given number be multiplied into itself once, it produces the square of that given number: as $3\times3=9$, which is the square of 3.

If a given number be multiplied into itself, and that product be again multiplied by the given number, it produces the cube of the given number: as $3\times3=9$*sq.*$\times3=27$, the cube of 3.

If a given number be multiplied into itself three successive times, it will give the biquadrate or fourth power: as $2\times2\times2\times2=16$: fourth or the second power, or square multiplied into itself, will give the fourth power: as $2\times2=4$*sq.*; then 4*sq.*$\times4=16$, the fourth power.

The sursolid or fifth power, is produced by multiplying a given number into itself 4 times successively; as $2\times2\times2\times2\times2=32$, the fifth power: or multiplying the cube by the square, will give the fifth power.

The sixth power, or cube cubed, is produced by 5 multiplications: or by squaring the cube: thus, $2\times2\times2\times2\times2\times2=64$: or 8×8 64.

The number denoting the power, is called the *index* or *exponent* of that power: as the square of $3=9^2$: the cube of $3=27^3$; and the biquadrate 81^4.

When two or more powers are multiplied together, the product is that power, whose index is the sum of the exponents of the factors.

$$\text{As } \overset{2}{4}\times\overset{2}{4}=\overset{4}{16}\text{; and } \overset{4}{16}\times\overset{4}{16}=\overset{8}{256}.$$

The names of the powers are more commonly distinguished by the numbers, 4, 5, 6, 7, 8, 9, 10, powers, &c.; the second power is usually called square; and the third cube; often the fourth biquadrate; and the fifth sursolid.

The given number is called the root; viz. the number which is squared, &c.

The following table will exhibit several powers of the 9 digits, except 1.

Roots.	2	3	4	5	6	7	8	9
Squares.	4	9	16	25	36	49	64	81
Cubes.	8	27	64	125	216	343	512	729
4th pow.	16	81	256	625	1296	2401	4096	[illegible]
5th pow.	32	243	1024	3125	7776	16807	32768	59049
6th pow.	64	729	4096	15625	46656	117649	262144	531441
7th pow.	128	2187	16384	78125	279936	823543	2097152	4782969

Note.—It will be seen, that multiplying the given number, root or first power, continually by itself, until the number of multiplications be 1 less than the index of the power to be found, and the last product will be the power required.

Vulgar Fractions are raised to the powers required, by multiplying their respective terms into each other, until they are raised to the power sought, or turning them into decimals. If a mixed number, it may be reduced either to an Improper Fraction; or to a Decimal Fraction.

Examples.

1. What is the square of 6? *Ans.* 36.
2. What is the cube of 7? *Ans.* 343.
3. What is the biquadrate or fourth power of 4?—*Ans.* 256.
4. What is the cube of 9? *Ans.* 729.
5. What is the sixth power of 6? *Ans.* 46656.
6. What is the seventh power of 7? *Ans.* 823543.
7. What is the fifth power of 8? *Ans.* 32768.
8. What is the square of 36? *Ans.* 1296.
9. What is the cube of $\frac{3}{9}$? *Ans.* $\frac{27}{729}$.
10. What is the square of $4\frac{5}{8}$?

$\frac{5}{8}$=,625=4,625 *Ans.* 21,390625.

Questions relative to Involution.

1. What is Involution?
2. How is a power produced?
3. If a given number were multiplied into itself once, what would it produce?
4. What power would be produced by multiplying a given number into itself, and that product again by the given number?
5. If a number be involved by three multiplications, what power is produced?
6. If one multiplication produce a square; two a cube; and 3 a biquadrate; what will 4, 5, or 6 multiplications produce?
7. What will a square multiplied into itself produce? and what would a cube produce?
8. What is meant by the term *index* or *exponent?*
9. When two or more powers are multiplied together, of what power is the product?
10. By what name is the given number called; viz. the number to be involved?
11. May Fractions, both Vulgar and Decimal, and also mixed numbers, be raised to powers, like whole numbers?

EVOLUTION;

OR THE EXTRACTION OF ROOTS.

THE root of any given *power*, is such a number, as being multiplied into itself a given number of times, will produce that *power*. Thus, 4 is the square root of 16, the *power* for $4\times4=16$: and 3 is the cube root of 27, the *power*; for $3\times3\times3=27$: so that any given *number* or *root*, when involved into itself a given number of times, produces the required *power*: consequently the *root* is the number which has been thus involved to produce this *power*.

Examples.

[Seen by the last table.]

1. What is the cube root of the power 512? *Ans.* 8.
2. What is the square root of the power 81? *Ans.* 9.
3. What is the biquadrate root of the power 1296?—*Ans.* 6.
4. What is the square root of the power 49? *Ans.* 7.
5. What is the root of the sixth power, viz. 46656?—*Ans.* 6.
6. What is the root of the seventh power 4782969?—*Ans.* 9.
7. What is the square root of the power 64? *Ans.* 8.
8. What is the sursolid root of the power 3125? *Ans.* 5.
9. What is the cube root of the power 512? *Ans.* 8.

Note.—Although there is no number but what will produce a perfect power by Involution; yet there are many numbers, of which precise roots cannot be exactly obtained. By the aid however of decimals, there may be an approximation towards the precise root, sufficiently near to answer the purposes intended.

The roots which approximate, but not with exactness, are called *surd* roots; but those which are perfectly accurate are denominated *rational* roots. The roots are often denoted by the characters placed before the power, with the index of the root over it; as $\sqrt[3]{}$ $\sqrt[4]{}$.

Questions relative to Evolution.

1. What is Evolution?
2. Is Evolution directly the reverse of Involution?
3. What is the *root* of a given power?
4. Is Evolution applicable to Fractions, equally with whole numbers?
5. Is there any number which will not produce a perfect square by Involution?
6. Is there any of which precise roots cannot be obtained by Evolution?
7. By what name are those roots called, which are perfectly obtained?
8. By what name are those called which cannot be precisely obtained?

EXTRACTION

OF THE SQUARE ROOT.

To extract the square root is to find such a number of a given power, which number being multiplied into itself, will give a product equal to the given power.

Rule.

1. Distinguish the given number or power into periods of two figures each, by putting a point over the place of units, then over hundreds, and so on over every second figure in the given power. If there be decimals, point them off from the unit's place, towards the right, by putting a point over the second figure, or place of hundredths of decimals, and every second figure. If the given decimals have not an even number of figures, annex a cipher. The number of points will distinguish the number of figures in the root when found: should there be a remainder, the operation may be continued at pleasure, by annexing two ciphers at a time.

2. Find the greatest square number in the first, or left-hand period, and place it under the period, and place the root of this square at the right-hand of the given power, (after the manner of the quotient in division,) for the first figure of the root; subtract the square from the period, and to the right of the remainder bring down the next period for a resolvend.

3. Double the quotient, and place it on the left of the resolvend for a divisor, (reserving always the unit's place in the divisor, which is yet to be supplied by the same figure, as is next placed in the quotient;) seek how often the divisor is contained in the dividend, (except the right-hand figure,) and put the answer in the quotient, and also on the right-hand of the divisor: then multiply the last quotient figure into the divisor, and subtract the product from the resolvend, bring down the next period to the remainder, and proceed as before.

Note.—The divisor is also found by bringing down the last divisor for a new one, doubling the right-hand figure of it.

Examples.

1. What is the square root of 50625?

```
           50625(225 Square root.
           4      2
           —      —
         42)106   44
            84
           ————
Dou. the rig.-hd. 455) 2225
 fi. of the last di. or 2225
 double the quo.        ————          Ans. 225.
```

2. What is the square root of 11943936?

```
          11943936(3456
          9
          —
       64)294
          256
          ———
      685)3839
          3425
          ————
     6906)41436
          41436
          —————                      Ans. 3456.
```

3. What is the square root of 10342656? *Ans.* 3216.
4. What is the square root of 234,09? *Ans.* 15,3.
5. What is the square root of ,054756? *Ans.* ,234.
6. What is the square root of 46? *Ans.* 6,7823+
7. What is the square root of 36372961? *Ans.* 6131+
8. What is the square root of 10? *Ans.* 3,162277+

To extract the Square Root of Vulgar Fractions.

Rule.—1. Reduce the fractions to the lowest terms, for this and all other roots. If a mixed number, reduce it to an improper fraction or a decimal.

2. Extract the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

3. If the fraction be a surd, reduce it to a decimal, and extract the root.

1. What is the square root of $\frac{64}{81}$? $\sqrt{\frac{64}{81}} \Big| \frac{8}{9}$ *Ans.* $\frac{8}{9}$.
2. What is the square root of $\frac{144}{225}$? *Ans.* $\frac{12}{15}$.
3. What is the square root of $27\frac{9}{16}$? *Ans.* $5\frac{1}{4}$.
4. What is the square root of $42\frac{1}{4}$? *Ans.* $6\frac{1}{2}$.
5. What is the square root of $\frac{36481}{145924}$? *Ans.* $\frac{191}{382}$.
6. What is the square root of $\frac{23}{45}$? *Ans.* ,7156+
7. What is the square root of $\frac{187}{215}$? *Ans.* ,9326+
8. What is the square root of $36\frac{1}{4}$? *Ans.* 6,0207+

Application and Use of the Square Root.

PROBLEM I.—1. Suppose an army, consisting of a certain number of men, were placed, rank and file, in the form of a square, each side having 475 men; how many would the whole square contain?

Rule.—Square the given number. *Ans.* 225625.

2. A certain pavement is made exactly square, each side of which contains 88 feet; how many square feet does the square contain? *Ans.* 7744.

PROB. II.—1. A General has an army of 6724 men; how many must he place in rank and file to form them into a square?

Rule.—Extract the square root. $\sqrt{6724}=82$ *Ans.*

2. The above square pavement contains 7744 square stones, all of the same size. How many are contained in one of its sides? $\sqrt[2]{7744}=88$ *Ans.*

PROB. III.—To form any body of soldiers, so that they may be double, triple, &c. as many in rank as in file.

Rule.—Extract the square root of ½, ⅓, ¼, &c. of the given number of men, and that will be the number of men in file; which double, triple, &c. as the question may require, and the product will be the number in rank.

1. Place 10368 men in such a form, that the number in rank shall be double the number in file.

$10368\div2=5184$; and $\sqrt[2]{5184}=72$ in file;
and $72\times2=144$ in rank.

2. Place 19683 men in such form, that the number in rank shall be triple the number in file.

$19683\div3=6561$; and $\sqrt[2]{6551}=81$ in file;
and $81\times3=243$ in rank.

PROB. IV.—To find a mean proportional between two numbers.

Rule.—Multiply the two numbers together, and extract the square root of the product.

1. What is the mean proportional between 14 and 56?

$56\times14=784$; and $\sqrt[2]{784}=28$ *Ans.*

2. What is the mean proportional between 15 and 60?

$60\times15=900$; and $\sqrt[2]{900}=30$ *Ans.*

PROB. V.—To find the area of a circle.

Rule.—Multiply the square of the diameter by ,7854, and the product will be the area. Or, multiply the square of the circumference by ,07958, and the product will be the area.

1. What is the area of a circle, whose diameter is 9?

$9\times9=81$; and $81\times{,}7854=63{,}6174$ *Area.*

2. What is the area of a circle, whose circumference is 32,5?

$32{,}5\times32{,}5=1056{,}25$; and $1056{,}25\times{,}07958=84{,}056375$ *Area.*

PROB. VI.—The area of a circle being given, to find the diameter?

Rule.—Multiply the square root of the area by 1,12732, and

the product will be the diameter. Or, multiply the area by 1,2732, and then extract the square root of the product, and the diameter is obtained.

1. What is the diameter of a circle whose area is 36,6174?

$$36,6174\times1,2732=80,99767368;$$
$$\text{and } \sqrt[2]{80,99767368}=8,9998 \textit{ Ans.}$$

Prob. VII.—The area of a circle given, to find the circumference.

Rule.—Multiply the given area by 12,566, and extract the square root of the product, and the circumference is obtained.

1. Suppose the area of a circle to be 160; what is the periphery, or circumference?

$$160\times12,566=2010,560;\ \sqrt[2]{2010,560}=44,84 \textit{ Ans.}$$

2. Suppose the area of a circle to be 113,03, what is the periphery?

$$113,03\times12,566=1420,33498;\ \text{the square root is }=37,68 \textit{ Ans.}$$

Prob. VIII.—The area of a circle given, to find the side of a square equal to the circle.

Rule.—Extract the square root of the area, and the answer will be the side of a square, equal in area to the circle.

Note.—The square root of the area of any figure whatever, is the side of a square equal to the given figure.

1. If the area of a circle be 625, what is the side of a square equal in area to the given circle?

$$\sqrt[2]{625}=25 \textit{ Ans.}$$

Prob. IX.—Suppose a farmer would set out an orchard of 600 trees, so that the length shall be to the breadth as 3 to 2, and the trees severally distant from each other 7 yards; how many trees will there be in length and breadth, and how many square yards of ground do they stand on?

In resolving any question of this kind,

Rule.—As the ratio in length is to the ratio in breadth, so is the number of trees to the fourth number; the square root of which will be the number of trees in breadth. And as the ratio in breadth is to the ratio in length, so is the number of trees to a fourth term; the square root of which is the number of trees in length.

As $3 : 2 :: 600 : 400$; and $\sqrt{400}=20$ trees in breadth.
$2 : 3 :: 600 : 900$; and $\sqrt{900}=30$ trees in length.

Then, as yd.

1 less from each number of trees $\left\{\begin{array}{l}1 : 7 :: 29 : 203 \\ 1 : 7 :: 19 : 133\end{array}\right\}$ $203\times133=2699$ *sq. yd.*

Ans. 20 trees in breadth, 30 trees in length; and 2699 square yards of ground.

Prob. X.—Suppose a cistern could be emptied by a pipe of 2 inches in diameter in 3 hours; what must be the diameter of a pipe to discharge 4 times as much water in the same time?

Rule.—Square the diameter, and multiply the square by the given proportion; and the square root of the product will be the answer.

$2\times2=4$
4 given proportion.
—
16

$\sqrt{16}=4$
Ans. 4 *in.* in diameter.

Prob. XI.—The sum of any two numbers, and their products being given, to find each number.

Rule.—From the square of their sum, subtract 4 times their product; then extract the square root of the remainder, which will be the difference of the two numbers: add half the difference to half the sum for the greater of the two numbers; and the half difference subtracted from half the sum, gives the less number.

1. The sum of two numbers is 65, and their product is 784: what are those two numbers?

The sum of the numbers $65\times65=4225$
The pro. of $784\times4=3136$

$\sqrt{10{\cdot}89}$(33
9
—
63)189
189
—

½ of 65=32½
½ of 33=16½
—
Greater 49
—

32½
16½
—
Less 16
—

Ans. 49 and 16 added are 65.
$49\times16=784$.

Prob. XII.—"In every right angled triangle, the square of the hypothenuse (or longest side) is equal to the sum of the squares of the two legs:" or the square root of the hypothenuse is equal to the square root of the sum of the squares of the two legs. *Pythagoras.*

If this were not so, carpenters could not square their frames by the measures of 3, 4, and 5; or 6, 8, and 10.

The two legs, viz. the base and perpendicular, given to find the hypothenuse.

1. The top of a fortress is 36 yards high, which is surrounded by a trench 49 yards wide; what must be the length of a ladder to reach from the outside of the ditch to the top of the fortress? *Ans.* 60,8+ yards.

2. A ladder, 75 feet in length, stands 45 feet from the base of a turret, and extends to the top of the same: how high is the turret?

Hypoth.	$75 \times 75 = 5625$	
Base	$45 \times 45 = 2025$	$\sqrt[2]{3600} = 60$ perpendicular.
	3600	*Ans.* 60 feet high.

Note.—In the last example, the square root of the difference of the square of the base and hypothenuse, is the height of the perpendicular. Had the hypothenuse and perpendicular been given, the square root of the difference of their squares, would give the base.

EXTRACTION

OF THE CUBE ROOT.

To extract the Cube Root, is to find a number, which being multiplied into itself, and then into that product, will produce the given number.

Rule.

1. Separate the given quantity into periods of three figures each, by putting a point over the unit figure, and every third figure from the place of units to the left; and if there be decimals, to the right.

2. Find the greatest cube in the left-hand period, and place its root in the quotient.

3. Subtract the cube thus found from the said period, and to the remainder bring down the next period for a resolvend.

4 Multiply the square of the quotient by 300, calling it the divisor.

5. Seek how often the divisor may be had in the resolvend, and place the result in the quotient: multiply the divisor by the last quotient figure, and place the product under the resolvend.

6. Then multiply the former quotient figure or figures by the square of the last quotient figure, and that product by 30, and place this product under the last. Under these two products place the cube of the last quotient figure, and the sum of these products call the subtrahend.

7. Subtract this subtrahend from the resolvend, and to the remainder bring down the next period for a new resolvend; with which proceed as before, until the whole is finished.

Examples.

1. What is the cube root of 74088?

```
                   74088(42
                   64
                   ----
4×4×300=4800)10088 First resolvend.
                  9600 )
  2×2×4=16×30=    480  }
        2×2×2=      8  )
                 -----
                 10088 First subtrahend. No remain.
                 -----
```

2. What is the cube root of 34965783? *Ans.* 327.

3. What is the cube root of 259694072? *Ans.* 638.

4. What is the cube root of 87528,384?

87528,384(44,4
64

$4\times4\times300=4800$)23528 First resolvend.

$\left.\begin{array}{r}19200\\ 4\times4=16\times64\times30=\ 1920\\ 64\end{array}\right\}$

21184 First subtrahend.

2344384 Second subtrahend.

580800)2344384. $44\times44\times300=580800$

$\left.\begin{array}{r}2323200\\ 21120\\ 64\end{array}\right\}$ $16\times44\times30$

2344384 Second Subtrahend.

Ans. 44,4.

5. What is the cube root of 166,375? *Ans.* 5,5.
6. What is the cube root of ,238328? *Ans.* ,62.

TO EXTRACT THE CUBE ROOT
OF VULGAR FRACTIONS.

Rule.

Reduce the fractions to their lowest terms; then extract the cube root of the numerator and denominator, for a new numerator and denominator. But if the fraction be a surd, reduce it to a decimal, and then extract the root.

Reduce also a mixed number to an improper fraction, and extract as above. If the quantity be a surd, reduce the fraction to a decimal.

Examples.

1. What is the cube root of $\frac{4096}{15625}$? *Ans.* ,638+
2. What is the cube root of $\frac{5}{8}$? *Ans.* ,979+
3. What is the cube root of $31\frac{15}{343}$? *Ans.* $3\frac{1}{7}$.
4. What is the cube root of $166\frac{3}{8}$? *Ans.* $5\frac{1}{2}$.

Questions in Application of the Cube Root.

1. Suppose a reservoir be so constructed, in a cubical form, that it is 10 feet long, 10 feet wide, and 10 feet deep; how many cubical feet will it contain?

$10\times10\times10=1000$ solid or cubical feet, *Ans.*

2. If a cubical piece of mahogany timber be 44 inches long, 44 inches broad, and 44 inches deep, how many cubic inches does it contain? *Ans.* 85144.

3. If a cubical piece of mahogany contain 85144 solid feet, what is the superficial content of one of its sides? *Ans.* 44 *ft.*

4. A statute bushel contains 2150,425 cubic inches. What will be the side of a cubic box containing the same quantity?

$\sqrt[3]{2150,425}=12,907$.

Note.—The solid contents of similar figures are proportioned to each other, as the cubes of their similar sides, or diameters.

4. If a bullet, 2 inches in diameter, weigh 5*lb.*; what will a bullet of the same metal weigh, the diameter of which is 4 inches?

$2\times2\times2=8$; $4\times4\times4=64$; as 8 : 5 : : 64 : 40 *Ans.*

5. If a globe of silver, of 4 inches in diameter, be worth \$250; what is the value of another globe of 8 inches in diameter?

$4\times4\times4=64$; $8\times8\times8=512$; as 64 : 250 : : 512 : \$2000 *Ans.*

The side of a cube being given, to find the side of a cube which shall be double, triple, &c. in quantity to the given cube.

Rule.—Cube the given side, and multiply it by the given proportion, and the cube root of the product will be the side sought.

6. If a cube of silver, whose side is 3 inches, be worth \$75; what will be the side of a cube of like silver, whose value will be 4 times as much?

$3\times3\times3=27$; $27\times4=108$; $\sqrt[3]{108}=4{,}61+$ *Ans.*

7. If a cubical vessel have one side of 4 feet, what will be the side of another cubical vessel, which shall contain 8 times as much?

$4\times4\times4=64$; $64\times8=512$; $\sqrt[3]{512}=8$ feet, *Ans.*

To find two mean proportionals between two given numbers.

Rule.—Divide the greater extreme by the less; and the cube root of the quotient, multiplied by the less extreme, will give the less mean: then, multiply the same cube root by the less mean, and the product will be the greater mean proportional.

8. What are the two mean proportionals between 8 and 216?

8)216(27; and $\sqrt[3]{27}=3$; $3\times8=24$; $24\times3=72$

Proof. As 8 : 24 : : 72 : 216. *Ans.* 24 and 72.

9. What are the two mean proportionals between 5 and 135?

$135\div5=27$; $\sqrt[3]{27}=3$; $3\times5=15$; and $3\times15=45$

Proof. As 5 : 15 : : 45 : 135. *Ans.* 15 and 45.

TO EXTRACT

THE BIQUADRATE ROOT,

OR FOURTH POWER.

First, extract the square root of the given number; then extract the square root of that square root, and the biquadrate root will be found.

Example.—The biquadrate power of $\sqrt{16}=\sqrt{4}=2$ *Ans.*

A GENERAL RULE FOR THE EXTRACTION OF THE ROOTS OF ALL POWERS.

Rule.

1. Prepare the given number for extraction, by pointing off from the units place as the required root directs.

2. Find the first figure of the root by trial, or by inspection into the table of powers, and subtract its power from the left hand period.

3. To the remainder bring down the first figure in the next period, and call it the resolvend.

4. Involve the root to the next inferior power to that which is given, and multiply it by the number denoting the given power, for a divisor.

5. Find how often the divisor may be had in the dividend, and annex it to the quotient.

6. Involve the whole root to the given power, and subtract it from the *given number*, as before.

7. Bring down the first figure of the next period to the remainder for a new resolvend; find a new divisor as before, and proceed in like manner until the whole is finished.

1. What is the biquadrate root of 244256?

$$
\begin{array}{rl}
 & 2\dot{4}425\dot{6}(22 \\
2\times2\times2\times2= & 16 \\
2\times2\times2=8\times4=32) & 84 \\
22\times22\times22\times22= & 244256
\end{array}
$$

Ans. 22 is the biquad. root.

2. What is the sursolid root of 281950621875? *Ans.* 195.
3. What is the cube root of 34965783? *Ans.* 327.
4. What is the square root of 10342656? *Ans* 3216.

Note.—The roots of the 4th, 6th, 8th, 9th, and 12th powers, may be obtained more readily thus:—

For the root of the 4th power.

Viz.	4th.—Extract the square root of the square root.	[*Exponents.* $\sqrt{\ }\times\sqrt[2]{\ }=\sqrt[4]{\ }$.
	6th.—Extract the square root of the cube root.	$3\times2=6$.
	8th.—Extract the square root of the fourth root.	$2\times4=8$.
	9th.—Extract the cube root of the cube root.	$3\times3=9$.
	12th.—Extract the cube root of the fourth root.	$3\times4=12$.

Questions relative to the Square, Cube, and Roots generally.

1. What is meant by the extraction of the Square Root?

2. What is the first step in preparing a given power for the extraction of its root?

3. If there be decimals in the given power, how are they pointed off?

4. If the decimal have not an even number of figures for pointing, what must be done?

5. What does the number of points exhibit?

6. Should there be a remainder, after all the periods are brought down, how then proceed?

7. What is the second step in this rule, in its various operations?

8. What is the third step? first, in finding a divisor? secondly, the quotient figure? thirdly, the subtrahend? and fourthly, a new resolvend?

9. How many, and what are the methods of finding a divisor, after the first divisor is obtained?

10. What is the rule for extracting the Square Root of Vulgar Fractions? also of a mixed number? and of a surd quantity?

11. How is the proof of the extraction of the Square Root?

12. What is meant by the extraction of the Cube Root?

13. What is the first step to prepare for the extraction of the Cube Root?

14. Does the number of points denote, as in Square Root, the number of figures the root will contain?

15. Are the places of decimals to be supplied, if necessary, to accommodate the points?

16. What is the second step? what the third? what the

fourth? what the fifth? what the sixth? and what the seventh step?

17. Should there be a remainder, after all the periods are brought down; how then proceed?

18. What is the rule for extracting the Cube Root of Vulgar Fractions?

19. If the fraction be a surd or mixed number, how proceed?

20. How is the biquadrate root extracted?

21. What is the general rule for extracting the root of all powers?

22. How may the roots of the fourth, sixth, eighth, ninth, and twelfth powers be more readily found?

23. How are the cube, and other roots proved?

ARITHMETICAL PROGRESSION.

There are five particulars to be observed in Arithmetical Progression, viz.:—

1. The first term.
2. The last term.
3. The number of terms.
4. The common difference
5. The sum of all the terms.

Any three of the foregoing being given, the other two may be found.

Problem I.

The first term, the last term, and the number of terms being given, to find the *sum* of all the terms.

Rule.

Multiply the sum of the extremes by the number of terms, and half the product will be the sum of the terms.

Examples.

1. The first term of an arithmetical series is 5, the last 53, and the number of terms are 9; what is the sum of the series.

53+5=58 The sum of the extremes.

Then 58×9÷2=261 *Ans.*

2. How many strokes does the hammer of a clock strike in 10 hours?

1+10=11 Sum of terms: 11×10÷2=55 *Ans.*

3. How many strokes of the hammer of a clock in 24 hours?

1+12=13: 13×24÷2=156 *Ans.*

4. A person purchased 26 yards of cloth, and gave for the first yard 6 cents, and for the last yard 164 cents: what did the cloth cost him? *Ans.* $22 10ct.

5. A man caused 200 oranges to be placed a rod distant from each other, in a right line, and a basket placed one rod from the first orange. He now offers them all to *Jimmy*, if he will fetch them one at a time and put them in the basket: how far must *Jimmy* travel to bring all of them singly to the basket?

Ans. 125 miles and 20 rods.

6. A merchant sold 100 yards of diaper, at 3 cents for the first yard, 6 for the second, 9 for the third, increasing 3 cents for every yard; what did the cloth amount to; and what was the average per yard?

Ans. Amount $151 50ct.: average per yard $1 51ct. 5m.

PROB. II.—The first term, the last term, and the number of terms given, to find the common difference.

RULE.

Divide the difference of the extremes by the number of terms less one, and the quotient will be the common difference.

1. The extremes are 5 and 53, and the number of terms are 9: what is the common difference?

53—5=48; and 48÷9—1=6 Common difference. *Ans.* 6.

2. A man had 11 sons; the youngest was 4 years old, and the eldest was 34; their ages differed alike: what was the common difference of their ages? *Ans.* 3 years.

3. A lot of goods is to be paid for at 12 instalments. The first payment is $5, and the last 60: what is the common difference in the payments; and how much is the whole amount?

Ans. Common difference $5: whole amount $390.

4. A pedestrian starts from New-York to travel to a given

place in 13 days, and is to go but 5 miles the first day, increasing each day by an equal excess, so as to make the last day's journey 65 miles: what is the common difference of his daily travel, and how great the whole distance?

Ans. Common difference 5 miles: whole distance 455 miles.

PROB. III.—Given the first term, last term, and common difference, to find the number of terms.

RULE.

Divide the difference of the extremes by the common difference, and the quotient increased by one, is the number of terms.

Examples.

1. Let the extremes be 5 and 53, and the common difference 6: what is the number of terms?

$53-5=48$; and $48\div6=8+1=9$ *Ans.*

2. Let the extremes be 5 and 60, and the common difference 5: what is the number of terms? *Ans.* 12.

3. Let the extremes be 3 and 65, and common difference 2: what is the number of terms? *Ans.* 32.

PROB. IV.—Given the last term, the number of terms, and the common difference, to find the first term.

RULE.

Multiply the number of terms less one, by the common difference, and that product subtracted from the last term, will give the first.

Examples.

1. Let the last term be 53, the number of terms 9, and the common difference 6: what is the first term?

$9-1\times6-53=5$ *Ans.*

2. The last term is 65, the number of terms 32, and common difference 2: what is the first term?

$32-1\times2=62$: and $62-65=3$ *Ans.*

PROB. V.—The first term, the number of terms, and common difference given, to find the last term.

Rule.

Multiply the number of terms by the common difference, and from that product subtract the common difference, and to the remainder add the first term, and it gives the last term.

Examples.

1. The number of terms is 32, the common difference 2, and the first term is 3; what is the last term?

$32 \times 2 - 2 = 62$; and $62 + 3 = 65$ *Ans.*

2. The number of terms is 9, the first term 5, and common difference 6; what is the last term? *Ans.* 53.

3. The number of terms is 12, the first term 5, and the common difference 5; what is the last term? *Ans.* 60.

Questions relative to Arithmetical Progression.

1. How many particulars are to be observed in Arithmetical Progression? and what are they?

2. How many of these particulars must be given, that the others may be found?

3. When the first term, the last term, and the number of terms are given, what is the rule to find the sum of all the terms?—*Prob. I.*

4. When the first term, the last term, and the number of terms are given, what is the rule to find the common difference?—*Prob. II.*

5. When the first term, the last term, and common difference are given, what is the rule to find the number of terms?—*Prob. III.*

6. When the last term, the number of terms, and common difference are given, what is the rule to find the first term?—*Prob. IV.*

7. When the first term, the number of terms, and common difference are given, what is the rule to find the last term?—*Prob. V.*

GEOMETRICAL PROGRESSION.

When any rank or series of numbers is increased by one common multiplier, or decreased by one common divisor, they belong to Geometrical Progression: as, 1, 2, 4, 8, 16, 32, are increased by the common multiplier 2; also, 81, 27, 9, 3, 1, decrease by the common divisor 3.

When any number of terms are in Geometrical Progression, the product of the two extremes will be equal to that of any two means equally distant from the extremes; and if the terms be odd, the middle term multiplied into itself will be equal to the product of the two extremes, or that of any two means equally distant from the middle term.

As, 2, 4, 8, 16, 32; Or, 1, 3, 9, 27, 81;

$$\left.\begin{array}{r}32\times2\\16\times4\\8\times8\end{array}\right\}=64 \qquad \left.\begin{array}{r}81\times1\\27\times3\\9\times9\end{array}\right\}=81$$

Five particulars are requisite equally in Geometrical Progression as in Arithmetical; viz.

1. The first term.
2. The last term.
3. The number of terms.
4. The common difference, or ratio.
5. The sum of all the terms.

Note.—To find the last term in a long series, by continued multiplications, is very burdensome. To find it therefore more readily, there is a series of numbers made use of in Arithmetical Progression, which is called *indices*, or *exponents*, whose common difference is one. Whatever number of indices are required by the question given, place as many numbers in such Geometrical Proportion as is required by the question directly under these Arithmetical *indices*, or *exponents*.

As, 1,	2,	3,	4,	5,	6,	Arithmetical indices, or exponents.
2,	4,	8,	16,	32,	64,	Geometrical series.

When the first term of the Geometrical Proportion and the ratio are alike, that is, of the same number, the Arithmetical indices commence with 1; and in this case, the *product* of any two terms in the Geometrical series, is equal to *that term* which is signified by the *sum* of their indices.

Thus, $\begin{cases} 1, 2, 3, 4, 5, 6, \text{ indices, or exponents.} \\ 2, 4, 8, 16, 32, 64, \text{ Geometrical series.} \end{cases}$

Now, add the indices, $4+6=10$, exponent, Arithmetical.
Geom. series under these, $16\times64=1024$, the 10th term, Geom.

Thus, by *adding* the indices, or exponents, to the required amount of a given term, and *multiplying* the Geometrical series under the exponents thus added, will give the same term in the required Geometrical series.

But if the first term in the Geometrical series and the ratio be different, viz. not one and the same number, then the indices must begin with a cipher.

As, $\begin{cases} 0, 1, 2, 3, 4, 5, 6, \text{ indices.} \\ 1, 2, 4, 8, 16, 32, 64, \text{ Geometrical.} \end{cases}$

In this and similar cases, the indices beginning with a cipher, the sum of the indices made use of must always be one less than the number of terms given in the question; for 1, in the indices, is over the second term, and 2 over the third term, &c.: and hence the product of any two terms is equal to that term beyond the first signified by the sum of their indices.

Thus, $\begin{cases} 0, 1, 2, 3, 4, 5 \text{ indices.} \\ 1, 2, 4, 8, 16, 32 \text{ Geometrical.} \end{cases}$

Here, $4+5$ of the indices would $=9$; but a cipher occupying the first place, leaves it less 1, viz. $=8$.

Then, the Geometrical series, viz. $16\times32=512$, the 9th term; but the 8th only beyond the first.

Thus, when the exponent 1 stands over the second term, the number of exponents must be 1 less than the number of terms. But if the exponent 1 stand over the first term, the number of the exponents must equal the number of terms.

Problem I.

The first term, the last term, and the ratio given, to find the sum of the series.

Rule.

Multiply the last term by the ratio, and from the product subtract the first term; then divide the remainder by the ratio less one, and the quotient will be the sum of all the terms.

Examples

1. If the terms of the series be 1, 3, 9, 27, 81, 243, 729, and the ratio 3: what is the sum of the series?

$729\times3-1=2186$: and $2186\div2=1093$ Sum of the series. *Ans.*

2. If the extremes of a geometrical series are 3 and 56789, and the ratio 3: what is the sum of all the terms?—*Ans.* 85182.

3. If the extremes of a geometrical series are 2, and 87653, and the ratio 4: what is the sum total of all the terms?

Ans. 116870

Prob. II.—The first term and ratio given to find any term required.

Case I.

When the first term of the series and the ratio are *equal*:

1. Write down a few leading terms of the series, and place indices over them, commencing the indices with unity or one

2. Then add together such indices, whose sum shall make up the entire index to the sum required.

3. Multiply the terms of the geometrical series, standing under the respective indices together, and the product will be the term sought.

Case II.

When the first term of the series and the ratio are *different*

1. Write down the leading terms of the series, and begin the indices with a cipher.

2. Add together such indices as are convenient to make an index less by 1, than the number expressing the place of the term sought.

3. Multiply the terms of the geometrical series together standing under the indices used, and the product will become a dividend.

4. Raise the first term to a power, whose index is one less

than the number of terms multiplied, and this will give a divisor, by which divide the last product, called a dividend, and the quotient is the term sought: or what is the same in effect. In multiplying the several terms of the geometrical series, recollect, that every product must be divided by the first term.

N. B.—This gives the last term. The sum total is found by Problem I.

Examples.

1. A draper sold 13 yards of broad cloth, at 3 farthings for the first yard, 9 for the second, and 27 for the third, &c. in a triple proportion geometrical; what was the last term; and what was the cost of the cloth?

1, 2, 3, 4 Indices.
3, 9, 27, 81 Geometrical series.

The indices 4+4+4+1=13
The geom. series under these indices used, are 81×81×81×3
=1594323 Farthings, 13th term.

Sum total, Prob. I. 1594323
Ratio. 3

Di. by ratio less 1, 2)4782969—3 First term.

4)2391483 Far. *Ans.* 1594323*qr.* 13 term.
or £1660 15*s.* 0*d.* 3*q.*

12) 597870=¾ Pence.

20)49822=6 Shillings.

2491=2 Pounds.

Amount £2491 2*s.* 6*d.* 3*qr.*

2. A drover took 9 horses into market, for which he was offered 200 guineas: this he refused, but agreed he would sell them on these terms; viz. for the first horse 5 farthings; for the second 25 farthings; for the third 125 farthings, &c., in a five fold ratio; to which the purchasers readily acceded: what did the ninth horse fetch; and what was the amount of the whole?

Ans. Ninth horse £2034 10*s.* 1*d.* 1*qr.*: whole amount £2543 2*s.* 7*d.* 1*qr.*

3. A schoolmaster offered to teach a school one year for $240

and his board. The employers thought it too great a salary. The teacher then proposed to receive only 2 shillings for the first month, 4 for the second, and 8 for the third, &c. in a two fold ratio geometrical, together with his board, to which they readily agreed: what salary did he receive?—*Ans.* £309 10*s.*

4. Nine individuals, some of whom were boys, drew a prize; and having ascertained, that if the youngest received $10, and the next youngest $40, and the third $160; and the whole prize were thus divided in a quadruple proportion geometrical, among the nine, according to their respective ages, the division would be satisfactory: how much did the last receive? and what the whole prize?

0 1 2 3 — 3+3+2=8 Number of terms less 1.
10, 40, 160, 640 — 640×640×160=65536000
Or 640×640÷10=40960 1st term 10×10=1|00)655360(00 $
40960×160÷10=655360 Dollars. *Ans.* $655360 The last re.
4 Ratio.

3)2621440—10

$ 873810 — Amount of the prize $873810.

5. If the first term of a geometrical series be 2, and the ratio 3; what will the tenth term be? and what the amount of all the terms?

0 1 2 3 4 — 4+4+1=9
2 6, 18 ,54 ,162 — 162×162×6=157464÷4=39366 Last term.
Or, 162×162÷2=13122
And 13122×6÷2=39366 Last term. 39366×3—2÷2=
Ans. 59053 Whole amount.

6. A gentleman married his daughter on New-Year's day, and he gave to her husband half a dollar towards her portion, promising to double it on the first day of every month during the year; what was her patrimony? *Ans.* $2730.

7. A person purchased a horse, newly shod all around, with 8 nails in each shoe, on these conditions, viz. he was to give 1 farthing for the first nail, 2 for the second, 4 for the third, and so on, in a duplicate ratio for each nail in the 4 shoes; what did the horse amount to? *Ans.* £4473924 5*s.* 3*d.* 3*qr.*

8. A merchant sold 20 yards of silk velvet, for which he paid

£4 per yard, on the following conditions: for the first yard 2 pins, for the second 8 pins, for the third 32 pins, &c. increasing in a fourfold proportion. Did he gain or lose by his bargain, allowing him to sell his pins at 20 for a penny; what did the 20th yard amount to; and what was the total value?

Ans.	He gained - - -	£229064842	9s.	0d.
	The 20th yard was	114532461	4	6
	The total amount -	229064922	9	0

Questions relative to Geometrical Progression.

1. When any rank or series of numbers is increased by a common multiplier, or decreased by a common divisor, to what do they belong?

2. When any number of terms are in Geometrical Proportion, how is the product of the extremes, or that of any two means equally distant from the extremes?

3. How many particulars are required in Geometrical Progression? and what are they?

4. What is that series of numbers called, belonging to Arithmetical Progression, whose common difference is one, and is placed over the terms of a series in Geometrical Progression, to find more readily the last Geometrical term required?

5. When the first term of the Geometrical series and the ratio are alike, with what *indices*, or *exponents*, do the Arithmetical series commence?

6. When the *indices* or exponents commence with 1, what is the product of the terms, in the Geometrical series, standing *under* the indices or exponents, added, equal to?

7. Does the *adding* of the indices, to make the required amount of the given term, and *multiplying* the Geometrical series standing under the indices added, produce the same term in the Geometrical series?

8. If the first term in the Geometrical series and the ratio be different, how do the indices commence?

9. When the indices begin with a cipher, what must be the sum of the indices used, to find a required term in the Geometrical series?

10. Why is the sum of the indices one less than the Geometrical terms required?

11. What is the rule, when the first term, the last term, and the ratio are given, to find the sum of the series?—*Prob. I.*

12. What is the rule, when the first term and the ratio are given, to find any other term?—*Prob. II. Case* 1.

13. What is the rule, when the first term of the series and the ratio are different?—*Case* 2.

14. When the last term of the series is found, how is the sum of the series obtained?

PERMUTATION OF QUANTITIES.

THIS rule shows how many different ways any given number of quantities or things may be varied in their positions, or changes.

RULE.

Multiply all the terms of the natural series of numbers, from one up to the given number, continually together, and the last product will be the answer required.

Examples.

1. Into how many different positions are the four letters at the beginning of the alphabet susceptible of changes?

$1\times2\times3\times4=24$. *Ans.* 24.

2. In how many different positions may 6 horses be harnessed before a carriage?

$1\times2\times3\times4\times5\times6=720$ *Ans.*

3. Ten students belonging to one class in the languages, being strongly attached to each other, and exceedingly ambitious, resolved they would not vacate the institution so long as they could sit in a different daily position at their recitations. How long must they, in such a case, be absent from their homes?

Ans. 9941 years and 335 days.

COMBINATION

Is a rule for discovering how many different ways a less number may be combined out of a greater.

RULE.

Find the product of a series from 1×2×3, &c. up to the given number to be combined; also, find the product of the descending series, from the given number out of which the combinations are to be made; divide one product by the other, and the quotient will be the answer.

Examples.

1. How many combinations of 6 letters out of 10?

Product of the series of 6 upwards, viz.

1×2×3×4×5×6=720, divisor.

Do. down from 10, 10×9×8×7×6×5=151200, dividend.

151200÷720=210 *Ans.*

2. How many combinations can be made of 4 letters out of 8?

Series of 4 upwards, viz. 1×2×3×4= 24

Do. of 4 downwards from 8, viz. 8×7×6×5=1680

1680÷24=70 *Ans.*

3. How many different half-dozens may be selected out of 20; and what will be the total value of all these half-dozens, at 6 cents for each half-dozen?

1×2×3×4×5×6= 720 divisor.

20×19×18×17×16×15=27907200÷720=38760÷6=6460,

6460 half-dozens,

at 6 cents 6

total value $387,60 *Ans.*

4. A drover bargained with a gentleman for 12 fat lambs, at $2 a-piece, which he was to select out of 24. Being long in choosing them, the offer was made to the drover, that for *one cent* for every different dozen which might be chosen out of the

24 lambs, he might have the whole; which was readily complied with. What must the drover pay? *Ans.* $37560 50ct.

Questions relative to Permutation and Combination.

1. What is Permutation?
2. What is the rule?
3. What is Combination?
4. What is the rule?

ANNUITIES OR PENSIONS

AT COMPOUND INTEREST.

Case I.

To find the amount of an Annuity or Pension in arrears (viz. forborne or unpaid,) at Compound Interest.

Rule.

1. Make 1 the first term of a geometrical progression; and the amount of $1, or £1, for one year, at the given rate per cent., the ratio.
2. Carry on the series to as many terms as the given number of years, and find the sum.
3. Multiply the sum thus found by the given annuity, and the product will be the amount sought.

Examples.

1. What will an annuity of $120 per annum, payable yearly, but forborne for 4 years, amount to at 6 per cent. Compound Interest?

[See tabular number, Table II., against 4 years.]

$1+1{,}06+(1{,}06)^2+(1{,}06)^3=4{,}374616$ the sum.
multiply by 120 annuity.

$524,953920 the amount sought.
Ans. $524 95ct. 3m.

The same result will follow by multiplying the tabular number in Table II., standing under the rate, and opposite the given time, by the annuity: as, tabular number 4,374616×120 =$524,95.3.

2. If a pension, or yearly salary of $180 be forborne 12 years, at 6 per cent. compound interest: what is the amount?

Tabular number 16,869942×180=$3036 59*ct.* nearly.

3. What will an annuity of £150, for 20 years in arrears, amount to, at 5 per cent. compound interest, annually

Tabular number at 5 per cent. for 20 years, 33,065954×150 =£4959,893100

```
                          20
                   ---------
Ans. £4959 17s. 10d. 1qr.   17,862000
                          12
                   ---------
                   10,344000
                           4
                   ---------
                    1,376000
                   ---------
```

4. What will a salary of $250, payable annually, but in arrears for 10 years, amount to, at 5 per cent. compound interest?

12,577892×250=$3144 47*ct.* 3*m.* *Ans.*

Case II.

To find the present worth of annuities at compound interest.

Rule.

Divide the annuity by that power of the ratio, signified by the number of years; subtract the quotient from the annuity divide the remainder by the ratio less 1, and the quotient will be the present worth.

Examples

1. What ready money will purchase an annuity of $150, to continue 6 years, at 6 per cent. compound interest?

1,06×1,06×1,06×1,06×1,06×1,06=1,41851, 6 pow. of the ra.

1,41851)150,00000000(105,74496÷150=44,2552397

44,2552397÷,06=$737 58*ct.* 7*m.* *Ans.*

Or, the answer is more readily obtained by Table III. Take the tabular number against 6 years, under the 6 per cent., which multiply by the annuity, and the present worth is obtained.

In the last example, the tabular number 4917324

150 Annuity.

737,598600

Ans. \$737 59*ct.* 8*m.*

2. What is the present worth of an annuity of \$500 per annum, to continue 15 years, at 5 per cent. compound interest?

Tabular number for 15 years, at 5 per cent. 10,379658×500 =\$5189 82*ct.* 9*m.*+ *Ans.*

3. What is the present worth of £200 per annum, to continue 12 years, at 6 per cent. compound interest?

Ans. £1676 15*s.* 4*d.* 2*qr.*

4. What is the present worth of \$45 per annum, to continue 30 years, at 5 per cent. compound interest?—*Ans.* \$691 76*ct.*+

Case III.

To find the present worth of annuities, &c. taken in *reversion*, at compound interest.

Note.—An annuity in *reversion* does not come into possession until a given time has elapsed; or, there is a given period of time before the annuity commences.

Rule.

Find the present worth of the annuity at compound interest, as in the last case; and the result will be the present worth, to commence immediately.

Divide this result by that power of the ratio denoted by the time of reversion, and the quotient will be the present worth of the annuity in reversion.

Examples.

1. What is the present worth of an annuity of \$200, not to commence until 6 years hence, and then to continue 3 years, at 6 per cent.

Annu.
The 6th power of 1,06=1,41854)200,00000000(140,993
Subtract the quotient 140,993

Divide by 1,06—1=,06)59,0070

Divide by 3d power of 1,06=1,19101)983,4500000(825,73.
Ans. $825 73*ct.*

The shorter by Table III.

Find the present value of $1 or £1, at the given rate, for the sum of the time of continuance and of reversion. From this amount subtract the present worth of $1 or £1, for the time in reversion. Multiply the difference by the annuity, and the product will be the answer.

In example first:

Time of continuance is 9 years, the tabular number is	6,801692
The time of re. 3*yr.* do. do.	2,673012
Difference	4,128680
Multiplied by the Annuity	200
	825,736000

Ans. $825 73*ct.* 6*m.*

2. What is the present worth of an annuity of $180, which is not to commence until 8 years have expired, and then to continue 4 years, at 6 per cent.?

Tabular number of 12 years=8,383844
Do. do. 4 do. =3,465106

4,918738×180=

Ans. $885 37*ct.* 2*m.*+

3. What is the present worth of a reversion of £175 per annum, to continue 18 years, but not to commence until the end of 12 years, at 5 per cent.?

15,372451
8,863252

6,509199×175=£1139 2*s.* 2*d.* 1*qr.*

4. What is the present worth of a reversion of \$300, yearly rent, to commence at the close of 12 years, and continue 12 years, at 6 per cent.? *Ans.* \$1249 95*ct.* 3*m.*+

Case IV.

To find the present worth of an annuity or freehold estate, to continue *for ever*, at compound interest.

Rule.

As the rate per cent. is to 100; so is the yearly rent to the value required.

Examples.

1. What is the value of a freehold estate of £60 per annum, allowing 6 per cent. to the purchaser?

	£		£		£		
As	6	:	100	: :	60	:	*Ans.* £1000.

2. What is the present worth of a yearly pension of \$200, allowing the purchaser 5 per cent. for ready money?

	\$		\$		\$		
As	5	:	100	: :	200	:	*Ans.* \$4000.

3. If an estate give yearly an income of \$500; how much would it sell for, allowing the purchaser 6 per cent. for money advanced? *Ans.* \$8333 33*ct.* 3*m.*+

Case V.

To find the present worth of an annuity or freehold estate in reversion, at compound interest.

Rule.

1. Find the present value by the last rule, as if it were to be entered upon immediately: then divide the value already found by that power of the ratio specified by the time of reversion, and the quotient will be the present worth of the estate in reversion.

Examples.

1. If an estate of \$200 per annum, to commence 2 years hence, be offered for sale: what is the value, allowing the purchaser 5 per cent.?

As 5 : 100 : : 200 : \$4000. And 4000÷1,025=
\$3628 11*ct.* 7*m.*

Ans. \$3628 11 7, is the present worth of \$4000, in 2*yr.* rever.

Or by Table III.

Find the present worth of the annual rent, for the time of reversion; subtract this from the value of the immediate possession, and it will leave the value of the estate in reversion.

In the last example, the tabular number, 1,859410, is the present worth of \$1 for 2 years.

1,859410
200 Annual rent.

371,882000—4000=\$3628 11*ct.* 8*m.* *Ans.*

2. Suppose an estate of \$300 per annum to commence 10 years hence, were offered for sale, allowing the purchaser 6 per cent.: what is it worth?

Tabular number for 10 years 7,360087, at 6 per cent.
300

6 : 100 : : 300 :=5000 2208,026100—5000=
\$2791 97*ct.* 3*m.* *Ans.*

3. Which is preferable, a term of 30 years in an estate of \$150 per annum; or the reversion of such an estate for ever, after the 30 years, at 6 per cent. per annum, compound interest?

	\$	Tabular number for
As 6 : 100 : : 150 :	2500	30 years=13,590721
Pre. worth of rent	2038,60815	150
	461,39185	=2038,608150
30 years is pref.	\$ 577,21630	*Ans.* \$577 21*ct.* 6*m.*

4. Which is most advantageous, a term of 14 years in an annuity of \$400; or the reversion of such an annuity for ever, after the 14 years, at 5 per cent. compound interest?

Ans. 14 years is preferable by \$81 08*ct.* 7*m.*

Questions relative to Annuities or Pensions, at Compound Interest.

1. When are Annuities or Pensions said to be in arrears?
2. What is the rule to find the amount of Annuities in arrears, at Compound Interest?
3. What is the rule to find the present worth of Annuities, at Compound Interest?
4. How is the present worth found by tabular numbers?
5. When is an Annuity said to be in reversion?
6. What is the rule to find the present worth of Annuities taken in reversion, at Compound Interest?
7. What is the rule to find the present worth of an Annuity or freehold estate, to continue *for ever*, at Compound Interest?
8. What is the rule to find the present worth of a freehold estate in reversion, at Compound Interest?

MENSURATION OF

SUPERFICES AND SOLIDS.

Section I.

OF SUPERFICES.

Superfices, often called area, is the *surface* of any body, the contents of which is found by the various measurements applied to different bodies. As a piece of land 40 rods long, and 4 rods wide, these dimensions multiplied together give 160 square rods =1 acre, the area, or superficial content of that ground. Also, a table-leaf, whose length is 4 feet, and breadth 3½ feet, give an area, or superficial content, of 14 square feet. Hence, the superfices or area of any body, is composed of

superficial squares, (not solid squares, involving depth,) which are larger or smaller, as different measurements are applied in mensuration; viz. rods, yards, feet, inches, &c. The surface of a board 12 inches in length and width, contains 144 square inches, or squares of 1 inch each.

The superficial content of any surface is readily found by the subsequent rules.

Article I.

To find the superficial contents of a square having equal sides.

Rule.—Multiply the side of the square into itself, and the product will be the area.

Examples.

1. How many square yards are contained in a carpet covering 6 yards square?

$6 \times 6 = 36$ square yards, *Ans.*

2. How many square inches in a surface 18 inches square?

$18 \times 18 = 324$ *Ans.*

3. How many square rods in a lot of land, which is 30 rods square? *Ans.* 900.

Article II.

To measure a parallelogram, or long square.

Rule.—Multiply the length by the breadth, and the product will be the area.

Examples.

1. A hall, in the form of a parallelogram, is 80 feet long and 60 wide; how many square feet are contained in it?

$80 \times 60 = 4800$ *Ans.*

2. How many square feet in a mahogany plank, 24 feet long, and 2½ feet wide? $24 \times 2\frac{1}{2} = 60$ *Ans.*

Note.—If the length be feet and the breadth inches, or vice versa, multiply them together, and divide the product by 12, and the quotient will be square feet. In the last example, 24 feet $\times$ 30 inches $\div 12 = 60$ feet, *Ans.*

If inches are multiplied by inches, divide by 144, and the quotient is feet. Hence, dividing 144 by the inches in breadth, and the quotient will be the length requisite to make a square foot.

Ex. 1.—If a board be 6 inches wide, how much in length will make a square foot?

144÷6=24 inches. Proof, 24×6=144 *sq. in.* =1 foot.

Ex. 2.—If a strip of land be 8 rods wide, how long must it be to make an acre?

160÷8=20 rods, *Ans.*

Article III.

To measure a rhombus, or rhomboides.

A rhombus resembles a square, pushed out of its original shape into the form of a diamond; and consequently contains two obtuse and two acute angles. A rhomboid is a parallelogram, apparently pushed out of its original shape, having its two opposite sides and opposite angles equal.

Rule.—In a rhombus, multiply one of the four equal sides, and in a rhomboides, one of the longest sides, into the perpendicular let fall from an obtuse angle to the side opposite in a rhombus, and one of the longest sides in a rhomboides, and the product will be the area.

Examples.

1. A rhombus has 4 equal sides of 16 feet each. A perpendicular let fall from one of the obtuse angles to its opposite side, measures 12 feet. What is the area of the rhombus?

16×12=192 area, *Ans.*

2. A rhomboides has two sides of 20 feet each; and a perpendicular let fall from an obtuse angle to one of its longest sides is 8 feet long: how many square feet does the area contain? 20×8=160 *Ans.*

Article IV.

To measure a triangle.

A triangle contains three sides and three angles; and is equal to half of a square, rhombus, rhomboides, &c.

Rule.—In a right-angled triangle, multiply the half of one leg of the triangle into the whole of the other leg, and the product will be the area: or, in an oblique-angled triangle, multiply half of the base, viz. the side on which the triangle rests, into the whole of the perpendicular let fall upon it from the opposite angle: and vice versa.

Examples.

1. In a right-angled triangle, let the base, or bottom leg, be 20, and the perpendicular, or other leg, be 16; what is the area of the triangle?

The half of $20=10$, and $10\times16=160$ *Ans.*
Or, the half of $16=8$, and $8\times20=160$ *Ans.*

2. In an oblique-angled triangle, let fall a perpendicular from the oblique angle upon the opposite or longest side, the length of which is 12, and the side on which it falls is 30 feet; what is the area of the triangle?

Half of $30=15$, and $15\times12=180$ *Ans.*
Or, half of $12=6$, and $6\times30=180$ *Ans.*

3. What is the area of a triangular piece of land, whose base is 36 rods, and the perpendicular height is 18 rods?

Half of $36=18$, and $18\times18=324$ *Ans.*
Or, half of $18=9$, and $9\times36=324$ *Ans.*

Article V.

To measure any irregular figure.

Rule.—Divide the figure into triangles, by drawing diagonal lines from one angle to another, which diagonals are taken from the same scale of equal parts as that from which the original figure was laid down; then measure all the triangles by the rules already given for the mensuration of triangles, and the sum of the several triangles will be the area of the given figure.

Article VI.

To measure any regular polygon.

A regular polygon is a figure, whose sides and angles are all

equal, and literally signifies many cornered; and their respective names are taken from the number of sides they severally contain, according to the names used in the Greek language, in counting:

Viz.—	3	Trigon.		11	Endecagon.
A figure having	4	Tetragon.	Also	12	Dodecagon.
Equal sides and angles is called	5	Pentagon.			
	6	Hexagon.			
	7	Heptagon.			
	8	Octagon.			
	9	Enneagon.			
	10	Decagon.			

Rule.—Multiply the length of one of the sides by the number of sides, and this product by the half of a perpendicular let fall from the centre of the figure to the middle of one of the sides, and the product will be the area of the polygon.

Examples.

1. In a hexagon or six sided figure, each side is 20 rods, and a perpendicular let fall on one of them, from the centre of the figure, is 8 rods: what is the area of the hexagon?

$20\times6\times4=480$ Square rods: and $480\div160=3$ Acres. *Ans.*

2. In a dodecagon or 12 sided figure, each side is 16 yards, and the perpendicular let fall is 6 yards: what is the area?

$16\times12\times3=576$ Square yards. *Ans.*

Article VII.

To measure a circle, and also various considerations appertaining to circles.

1. The diameter of a circle given to find the circumference.

Rule.—As 113 is to 355; or more accurately by decimals; as 1 is to 3, 1416; so is the diameter of the given circle to the circumference: or, which is the same in effect, multiply the diameter by 3,1416, and the product is the circumference.

Examples.

1. There is a circle whose diameter is 16; what is the circumference?

As 113 : 355 : : 16 : $355\times16\div113=50{,}265+$ *Ans.*

Or, $16\times3{,}1416=50{,}265+$ *Ans.*

Also, di. the diam. by ,31831, viz. $16\div{,}31831=50{,}265+$ *Ans.*

2. The circumference of the circle given to find the diameter.

Rule.—As 355 is to 113; or, as 1 is to ,31831, so is the circumference to the diameter.

Ex.—The circumference of a circle is 50,2656: what is the diameter?

355 : 113 : : 50,2656 : Then 50,2656×113÷355=16+ *Ans.*
Or 1 : ,31831 : : 50,2656 : Then 50,2656×,31831=16+ Dia.

3. To find the area of a circle

Rule.—Multiply half the diameter by half of the circumference, and the product will be the area: or, multiply the square of the diameter by ,7854, (without finding the circumference,) and the product is the area.

Ex.—The diameter of a circle is 16, and the circumference is 50,2656: what is the area?

½ of 16=8

½ of 50,2656=25,1328×8=201,0624 Area. *Ans.*
Or, 16×16=256: and 256×,7854=201,0624 Area. *Ans.*

4. The circumference of the circle given to find the area, without finding the diameter.

Rule.—Multiply the square of the circumference by ,07958, and the product will be the area.

Ex.—The circumference of a circle is 50,2656: what is the area?

50,2656×50,2656×,07958=201,06 258640+ Area. *Ans.*

5. The area of a circle given to find the diameter.

Rule.—Multiply the given area by 1,2732, and the product will be the square of the diameter.

Ex.—The area of a circle is 201,0624: what is the diameter?
201,0624×1,2732=256, whole number. $\sqrt{256}$=16 Di. *Ans.*

6. The area of a circle given to find the circumference.

Rule.—Multiply the given area by 12,566, and extract the square root of the product; the result is the circumference.

Ex.—The area of a circle is 201,0624: what is the circumference?

201,0624×12,566=2526,5501184, the square root is 50,265+ Nearly. *Ans.*

7. The diameter of a circle given to find the side of an equal square.

Rule.—Multiply the diameter by ,886227, or divide the diameter by 1,12838, and the quotient is the side of an equal square.

Ex.—The diameter of a circle is 16, what is the side of an equal square?

$16 \times ,886227 = 14,179632 = 14,179632$ *Ans.*

$16 \div 1,12838 = 14,179+$ The side of an equal square. *Ans.*

8. The diameter of a circle given, to find the side of an equilateral (viz. equal sided) triangle inscribed.

Rule.—Multiply the diameter by ,866024, or divide by 1,1547, and the side of an equilateral triangle is obtained.

Ex.—The diameter is 16; what is the side?

$16 \times ,866024 = 13,856384$ Side. *Ans.*

Or $16 \div 1,1547 = 13,8563+$ *Ans.*

9. The diameter of a circle given, to find the side of a square inscribed.

Rule.—Multiply the diameter by ,707016, or divide it by 1,414213, and the side is found.

Ex.—The diameter is 16; what is the side of an inscribed square?

$16 \times ,707016 = 11,312256,$

$16 \div 1,414213 = 11,31+$ length of side, *Ans.*

10. The circumference of a circle given, to find the side of a square which is equal.

Rule.—Multiply the circumference by ,282094, or divide it by 3,544907; the result is the length of the side.

Ex.—The circumference is 50,2656; what is the length of the side?

$50,2656 \times ,282094 = 14,1796+$ side, *Ans.*

11. The circumference of a circle given, to find the side of an equilateral triangle inscribed.

Rule.—Multiply the circumference by ,2756646, or divide it by 3,6275939, and the side is found.

Ex.—The circumference is 50,2656; what is the length of the side?

50,2656×,2756646=13,856+, *Ans.*

12. The circumference of a circle given, to find the side of a square inscribed.

Rule.—Multiply the circumference by ,225079, or divide it by 4,442877, and the side of the inscribed square is found.

Ex.—The circumference is 50,2656; what is the length of the side?

50,2656×,225079=11,313+, *Ans.*

13. The side of a square given, to find the diameter of a circle equal to the square whose side is given.

Rule.—Multiply the given side by 1,129, or divide it by ,886, and the result will be the diameter of a circle, the area of which is equal to the area of the given square.

Ex.—The side of a square is 14,1796; what is the length of the diameter, the area of whose circle shall equal the area of the square of which the side is taken?

14,1796×1,129=16,00+, *Ans.*

14. The side of a square given, to find the circumference of a circle equal to the given square.

Rule.—Multiply the given side by 3,545, or divide it by ,282, and the result will be the circumference.

Ex.—The side of the square is 14,1796; what is the circumference?

14,1796×3,545=50,2666+, *Ans.*

Note.—The area of a semicircle is found by halving the area of a whole circle of the same diameter. A quadrant, or fourth part of a circle, in a similar manner.

Article VIII.

To find the area of an ellipsis.

Definition.—An ellipsis, or ellipse, is not a perfect circle, but is of an oval form; and has two diameters, one longer than the other: the longer is called the transverse, and the shorter the conjugate diameters.

Rule.—Multiply the two diameters of the ellipsis together, and that product by ,7854: the last product will be the area.

Ex.—The transverse diameter of an ellipsis is 66, and the conjugate diameter is 60; what is the area of the ellipsis?

$66 \times 60 \times ,7854 = 3110,1840$ *Ans.*

Section II.

OF SOLIDS.

Solids consist of length, breadth, and depth, or thickness; and therefore embrace one property more than a superfices. Solids involve in mensuration cubic measure; as cubic inches, feet, yards, &c.

A *cube* is a solid of 6 equal sides, each of which is an exact square. Hence, a solid or cubic foot contains 1728 inches; or $12 \times 12 \times 12 = 1728$ square inches.

Article IX.

To measure a cube.

Rule.—Multiply one side into itself, and that product by the same side, and the last product will be the solid content of the cube. In other words, cube one of its sides, and the content is found.

Ex. 1.—The side of a cube is 20 inches; what is the solid content?

$20 \times 20 \times 20 = 8000$; and $8000 \div 1728 = 4$*ft.* 1088*in.* *Ans.*

Or, by decimals, $8000,0000 \div 1728 = 4$*ft.* 6296+*in.* *Ans.*

Ex. 2.—If a cistern be dug 10 feet square and 10 feet deep, how many cubic feet of earth must be thrown out?

$10 \times 10 \times 10 = 1000$ cubic feet, *Ans.*

Article X.

To measure a parallelopipedon.

Definition.—A parallelopipedon is a solid of three dimen-

sions, viz. length, breadth, and thickness; somewhat like the form of a chest, if solid; or a piece of timber exactly squared, but the length of which is much greater than the breadth or thickness. The ends are called bases, which are equal.

Rule.—Find the area of the base, and multiply that product by the length, which will give the solid content.

Ex.—The base is 18 inches square, and the length is $12\frac{1}{2}$ *ft.*; what is the solid content?

in. in. in.
$18\times18\times150=48600$; and $48600,000\div1728=28,125$ *Ans.*
By decimals, $1,5\times1,5\times12,5=28,125$ *Ans.*
Or, $18\times18\times12\frac{1}{2}=4050$; and $4050,000\div144=28,125$ *Ans.*

Note.—If a piece of timber, or any material, be of an equal bigness its whole length, although the breadth and thickness are different; yet if the breadth and thickness be multiplied together, and that product by the length; the last product will be the solid content.

Ex.—If a stick of timber be at the base 18 by 14 inches, and the length 16 feet: what is the solid content?

$18\times14\times16=4032$: and $4032\div144=28$ *Ans.*
Or by duodecimals; $1,6\times1,2\times16=28$ *Ans.*

Note.—If the three dimensions given be all in inches, the last product must be divided by 1728. If two of the dimensions only be inches, and one in feet, divide by 144. If one only be inches, divide by 12. If all three be feet, no division is required.

Article XI.

When the side of a square solid is given in inches, to find how much in length will make a solid foot.

Rule.—Divide 1728 by the area of the base or end, and the quotient will be the length required to make a solid foot.

Ex.—If the base of a piece of timber or marble, be six inches square, what length will be requisite to constitute a solid foot?

$6\times6\div1728=48$ *Ans.* 48 inches, or 4 feet long.

Note.—When two sides of an unequal square solid (viz. of unequal breadth and depth) are given, to find what length, in feet, will make any number of solid feet:

Rule.—Multiply any supposed number of feet by 144, and divide this product by the product of the breadth and depth, the quotient will be the length in feet.

Ex.—Suppose 24 feet in length be taken from a piece of timber, which is 14 inches wide and 10 inches deep: what will be the real length found?

$$\text{The supposed } 24\times144\div\overline{14\times10}=\overset{ft.}{24,685}+ \ \textit{Ans.}$$

ARTICLE XII.

To measure a cylinder.

A cylinder is a round body, resembling a column, whose bases are circles of equal diameters.

Rule.—Multiply the square of the diameter of one base, by ,7854, to find the area of that base; and the area thus found multiply by the length of the cylinder; and the last product will be the solid content.

Ex.—The diameter of a cylinder is 18 inches, and its length is 12½ feet: what is the solid content of the cylinder?

$$\text{Decimally; } 1,5\times1,5\times,7854\times12,5=\overset{so.\,ft.}{20,089375} \ \textit{Ans.}$$
$$\text{Or thus: } 18\times18\times12\tfrac{1}{2}\times,7854\div144=20,089375 \ \textit{Ans.}$$

Note.—Divide the solidity of a cylinder by the square of the diameter multiplied by ,7854, and the quotient will be the length.

Ex.—The solidity of a cylinder is 20,0893750, and its diameter 18 inches: what is the length of the cylinder?

$$20,0893750\div\overline{1,5\times1,5\times,7854}=12,5 \text{ feet. } \textit{Ans.}$$

ARTICLE XIII.

To find how many solid feet of timber, a round stick, equally thick from end to end, will contain, when hewn square.

Rule.—Multiply twice the square of the semidiameter in inches, by the length in feet, divide the product by 144, and the quotient will be the answer.

Ex.—If the diameter of a round stick of timber be 24 inches, and its length 18 feet; how many solid feet will it contain when

hewn square; and what will be the content of the slabs and chips which are thus hewn from the square?

$12\times12\times2\times18\div144=36$, the solidity when squared. *Ans.*

And $24\times24\times,7854\times18\div144=56,5487+$, the total solidity

Subtract the square solidity	36	
Leaves the solidity of chips	20,5487	*Ans.* 20,5487.

Article XIV.

To find how many feet of boards, being square edged, of any given thickness, can be sawn from a log of any given diameter.

Rule.—Having the content of the square timber, say, as the thickness of the board, including the saw calf, is to the solid feet of the timber, so is 12 inches to the number of feet of boards.

Ex.—What is the number of feet of square edged boards $1\frac{1}{2}$ inches thick, including the saw calf, can be cut from a log 24 feet long, and 20 inches in diameter?

$10\times10\times2\times24\div144=\overset{ft.}{33,333}$ Solid content. *ft.*

Then, as $1,5 : 33,333 :: 12 :$ and $33,333\times12\div1,5=266,666$

Article XV.

The dimensions of any box, bin, &c. being given, to find how many bushels it will contain.

Note.—A bushel contains 2150,425 cubic inches.

Rule.—Divide the cubic inches contained by the box, bin, &c. by the cubic inches contained in a bushel, viz. 2150,425, and the quotient will give the number of bushels.

Ex.—A box or bin is 72 inches long, 60 wide, and 54 deep; how many bushels of wheat will it contain?

$72\times60\times54\div2150,425=108,4777$ *Ans.* 108,477 Bushels.

Article XVI.

To find the quantity of bricks required to build the walls of a house, the dimensions of the walls and also of the bricks being given.

Rule.—Find the sum of the length of the walls, from which subtract 4 times the thickness of the walls, for the allowance of the corners. Multiply the remainder by the height, and the product by the thickness of the wall: the last product will be the solid content of the whole wall, which being multiplied by the number of bricks contained in a solid foot, will give the total number required.

Ex.—What number of bricks, 8 inches long, 4 wide, and 2½ thick, are required to build the walls of a house 46 by 40 feet, 20 feet high, and one foot thick?

$8\times4\times2{,}5=80$; and $1728\div80=21{,}6$ bricks in a solid foot.

Then, $46+46+40+40=172$ feet, whole length of wall.

4 subtracted for the lap of corners.

$168\times20\times1\times21{,}6=725760$ bricks *Ans.*

Article XVII.

To ascertain the solidity of a brush heap, the dimensions of which cannot be taken by common mensuration.

Rule.—Take any vessel, the solid contents of which are readily calculated; into which place the brush, and then pour in as much water as will exactly cover the brush. Measure from the top of the vessel to the surface of the water; and having taken out the brush, again measure and find the difference by the fall of the water. This difference will show the solidity occupied by the brush, 1728 inches being a solid foot.

Article XVIII.

To find a ship's tonnage.

For single-decked Vessels.

Rule.—Multiply the length by the breadth of the main beam, and that product by the depth of the hold, and divide the last product by 95.

Ex.—What is the tonnage of a single-decked vessel, whose keel is 80 feet, beam 26, and depth 13 feet?

$80\times26\times13\div95=284\frac{60}{95}$ tons, *Ans.*

What will the above tonnage amount to at $12 per ton?

Ans. $3415,578+

Note.—When it is required that the deck be bolted at any height above the wale, it is customary to pay for half of the additional height to which the deck is thus raised; viz. one-half the difference is added to the former given depth, in calculating the tonnage.

Ex.—A shipper contracted for a single-decked vessel, of the dimensions as in the last example; but afterwards requires the deck to be laid 15 feet hold: what is the tonnage to be paid for?

$\frac{1}{2}$ of $2=1$ added to $13=14$.

$80\times26\times14\div95=306\frac{50}{95}$ tons, *Ans.*

For a double-decked Vessel.

Rule.—Take half the breadth of the main beam for the depth of the hold; and then proceed as in a single-decked vessel.

Ex.—Required the tonnage of a double-decked vessel, 72 feet keel, and 24 feet beam.

$72\times24\times12\div95=218\frac{26}{95}$ tons, *Ans.*

What will the tonnage amount to at $13 per ton?

Ans. $2837,557+

Note.—Divide the continued product of the length, breadth, and hold, in feet, by 100 for ships of war, and by 95 for merchant ships.

To find the Government Tonnage.

"For a double-decked vessel, take the length from the fore-part of the main stem to the after-part of the stern post, above the upper deck: take the breadth at the broadest part above the main wales, half of which breadth shall be accounted the depth of such vessel, and then deduct from the length three-fifths of the breadth; multiply the remainder by the breadth, and the product by the depth, and divide the last product by 95, the quotient of which shall be deemed the true contents or tonnage of such ship or vessel."

"If such ship or vessel be single-decked, take the length and breadth as above directed; deduct from the said length three-fifths of the breadth, and take the depth from the under side of the deck plank to the ceiling in the hold; then, multiply

and divide as above, and the quotient will be deemed the tonnage."

Article XIX.

The proof of any cable being found, to find the strength of any other cable.—The strength of cables, and consequently the weights of anchors, are as the cubes of their circumferences.

Rule.—As the cube of the circumference of any cable, is to the weight of its anchor; so is the cube of the circumference of any other cable, to the weight of its anchor.

Ex.—Suppose a cable 12 inches about require an anchor of 18 *cwt.*; what will be the weight of an anchor for a cable 8 inches about?

in. *cwt.* *in.*

As $12\times12\times12=1728$: 18 : : $8\times8\times8=512$:

Then, $512\times18\div1728=5{,}333+$ *cwt.* *Ans.*

Again, if a cable of 8 inches about require an anchor of 5,333+ *cwt.*; what must be the weight of an anchor for a cable 12 inches about?

As $8\times8\times8$: $5\frac{1}{3}$: : $12\times12\times12$: 18 *cwt.* *Ans.*

Article XX.

The dimensions of two similar built vessels of different capacities, and the burthen of one of them being given, to find the burthen of the other.

Rule.—The burthen of similar built vessels are to each other, as the cubes of their similar dimensions.

Ex.—If a length of keel of 75 feet give 300 tons burthen; what burthen will another give, whose keel is 80 feet long?

	tons.		*tons.*	*cwt.*	*qr.*	*lb.*	
$75\times75\times75$:	300 : :	$80\times80\times80$:	364	1	3	3+	*Ans.*

Again, if a length of keel of 90 feet give 518 tons and 8 *cwt.* burthen; what burthen will another give, whose keel is 75 feet long?

	tons.	*cwt.*		
$90\times90\times90$:	518	8 : :	$75\times75\times75$:	300 tons, *Ans.*

A CORDAGE TABLE,

Specifying the number of fathoms, feet, and inches, of a rope of any size, from 1 to 14 inches in circumference, to make a hundredweight.

Inches circumfer.	Fathoms.	Feet.	Inches.	Inches circumfer.	Fathoms.	Feet.	Inches.	Inches circumfer.	Fathoms.	Feet.	Inches.	Inches circumfer.	Fathoms.	Feet.	Inches.
1	486	0	0	4¼	26	5	3	7½	8	4	0	10¾	4	1	8
1¼	313	3	0	4½	24	0	0	7¾	8	3	6	11	4	0	3
1½	216	3	0	4¾	21	3	0	8	7	3	6	11¼	3	5	7
1¾	159	3	0	5	19	3	0	8¼	7	0	8	11½	3	4	1
2	124	3	0	5¼	17	4	0	8½	6	4	3	11¾	3	3	3
2¼	96	2	0	5½	16	1	0	8¾	6	2	1	12	3	2	3
2½	77	3	0	5¾	14	4	6	9	6	0	0	12¼	3	2	1
2¾	65	4	0	6	13	3	0	9¼	5	4	0	12½	3	2	0
3	54	0	0	6¼	12	2	0	9½	5	2	0	12¾	2	7	8
3¼	45	5	2	6½	11	3	0	9¾	5	0	6	13	2	5	3
3½	39	3	0	6¾	10	4	0	10	4	5	0	13¼	2	4	9
3¾	34	3	9	7	9	5	6	10¼	4	4	1	13½	2	4	0
4	30	1	6	7¼	9	1	6	10½	4	2	2	13¾	2	3	6
												14	2	2	1

Note.—The left-hand column is inches in circumference, and quarters; and then follow fathoms, feet, and inches in the succeeding column; and thus alternately through the several columns, each exhibiting what quantity of any required inches in the given table, would make a hundredweight.

Ex.—How much in length of 8½ inch rope would make a hundredweight?

In the fifth column, marked inches, is 8½, and against it in the sixth column, stand 6, 4, 3, which gives the required length of rope for one hundredweight, viz. 6 fathoms, 4 feet, and 3 inches.

A TABLE

Specifying the weight of any cable or rope of 120 fathoms in length, for every half inch, from 3 to 24 inches in circumference.

In. cir.	Cwt.	Qrs.	In. cir.	Cwt.	Qrs.	In. cir.	Cwt.	Qrs.	In. cir.	Cwt.	Qrs.	In. cir.	Cwt.	Qrs.
3	2	1	7	12	1	11	30	1	15½	60	0	20	100	0
3½	3	0	7½	14	0	11½	33	0	16	64	0	20½	105	0
4	4	0	8	16	0	12	36	0	16½	68	0	21	110	1
4½	5	0	8½	18	0	12½	39	0	17	72	1	21½	115	2
5	6	1	9	20	1	13	42	1	17½	76	2	22	121	0
5½	7	2	9½	22	2	13½	45	2	18	81	0	22½	126	2
6	9	0	10	25	0	14	49	0	18½	85	2	23	132	1
6½	10	2	10½	27	2	14½	52	2	19	90	1	23½	138	0
						15	56	1	19½	95	0	24	144	0

In the last table, in the columns marked inches, are given the circumferences of a rope or cable up to 24; and in each adjacent column their several respective weights in hundredweights and quarters.

Ex.—If a cable were 18½ inches in circumference, look in the column of inches for 18½; and in the adjacent column against it stand 85, 2, viz. 85 hundredweight, 2 quarters, the weight of a cable of 120 fathoms, 18½ inches in circumference.

Suppose a ship in a gale brought to an anchor; but the gale still increasing, it is deemed the most prudent to cut the cables and run: the consequence is, that 90 fathoms of 15½ inch, and 75 fathoms of 12 inch cables are lost: what must they be valued at, new cordage being $15 per hundredweight?

120 fath. of 15½in. cable, 60cwt.				120 fath. of 12in. cable, 36cwt.			
½=60	do.	do.	30	½=60	do.	do.	18
¼=30	do.	do.	15	¼=15	do.	do.	4.5
90			45	75			22.5
75	12		22 5	67.5×$15=$1012,50			
				⅓ deduct for new			337,50
			67. 5				$ 675.00 *Ans.*

CASK GAUGING.

By actual mensuration, the following rule is deemed preferable.

Rule.

1. Take the following dimensions of the cask in inches; viz. the diameter at the bung and the head, and take the length.

Note.—Measure the length of the stave, then the depth of the chimes and the thickness of the heads, subtract from the whole length, which leaves the length within the cask. Remember to take the head diameter close to its outside; and for small casks add ,3 of an inch; for casks of 30, 40, or 50 gallons, add ,4; and larger, and very large casks, ,5 or ,6 tenths added to the head diameter, will be very nearly the head diameter within. In taking the bung diameter, observe, by moving the rod backward and forward, whether the stave opposite to the bung be thicker or thinner than the rest, and allow accordingly.

2. Subtract the head from the bung diameter, and note the difference.

3. If the cask be much bulging *between the bung and the head*, multiply the difference by ,7; if not greatly curved, by ,65; if yet less, by ,6; and if nearly straight by ,55; and add this product to the head diameter, which reduces the cask to a cylinder.

4. Square the mean diameter thus found, and multiply the product by the length: divide the product by 231, (for wine gallons,) and the quotient is the number of gallons.

Note.—Wine, spirits, ale, beer, milk, &c. all come under the same standard in the United States, of liquid measure; viz. 231 cubic inches in a gallon: water is 282 cubic inches.

Ex.—Required the content in wine gallons of a cask, the bung diameter of which is 36 inches, head diameter is 28, and the length is 44 inches.

Bung diameter	36	28	
Head do.	28	5,6	added to head diam.
Difference	8	33,6	mean diameter.
Bulge	,7		
	5,6		

$33,6 \times 33,6 \times 44 \div 231 = 215,04$ *gal.* *Ans.*

2. To gauge a tub, cistern, or any vessel, in which one diameter is less than the other, to make it more convenient for hooping.

Rule.—Take the two diameters, and also the height or length, in inches: multiply the two diameters together, and to their product add one-third of the square of their differences, viz. the difference of the two diameters: then multiply this sum by the height or length of the vessel; the last product divide by 231 for wine gallons, or 282 for water gallons, and the quotient is the content.

Ex.—Suppose a tub is 44 inches diameter within, at the top, 38 inches within, at the bottom, and the perpendicular height is 50 inches; what is the content in wine gallons, and also in water gallons?

Top diameter 44
Bottom do. 38

$44 \times 38 = 1672$

Difference $6 \times 6 = 36$, $\frac{1}{3}$ of which is 12

1684

Then, $1684 \times 50 \div 231 = 364\frac{1}{2}$ gallons, nearly, wine.
And $1684 \times 50 \div 282 = 298\frac{164}{282}$ water.

Note.—The contents of any vessel being found in cubic inches, the number of feet, gallons, bushels, &c. is readily obtained by the following divisors.

$$\text{Divide by} \left\{ \begin{array}{c} 1728 \\ 231 \\ 282 \\ 2150,425 \end{array} \right\} \text{the quotient will be} \left\{ \begin{array}{l} \text{Cubic feet.} \\ \text{Wine gallons.} \\ \text{Water gallons.} \\ \text{Bushels.} \end{array} \right.$$

The system of practical gauging is greatly facilitated by the construction of a rod, which is graduated in such just proportions, on the principles of mensuration which are applied in cask-gauging, as that by placing the rod into a cask in different positions, the contents of the cask may be readily discovered on the graduated side of the rod. Both the contents of a cask and the wantage, if it is not full, are easily obtained.

TABLE I.—*Showing the Amount of $1 or £1, at Compound Interest.*

Yrs.	4 per cent.	Yrs.	5 per cent.	Yrs.	6 per cent.
1	1,040000	1	1,050000	1	1,060000
2	1,081600	2	1,102500	2	1,123600
3	1,124864	3	1,157625	3	1,191016
4	1,169859	4	1,215506	4	1,262477
5	1,216658	5	1,276282	5	1,338226
6	1,265319	6	1,340096	6	1,418519
7	1,315932	7	1,407100	7	1,503630
8	1,368569	8	1,477455	8	1,593848
9	1,423321	9	1,551328	9	1,689479
10	1,480244	10	1,628895	10	1,790848
11	1,539454	11	1,710339	11	1,898299
12	1,601032	12	1,795856	12	2,012196
13	1,665074	13	1,885649	13	2,132928
14	1,731676	14	1,979932	14	2,260904
15	1,800944	15	2,078928	15	2,396558
16	1,872981	16	2,182875	16	2,540352
17	1,947900	17	2,292018	17	2,692773
18	2,025817	18	2,406619	18	2,854339
19	2,106849	19	2,526950	19	3,025600
20	2,191123	20	2,653298	20	3,207135
21	2,278768	21	2,785963	21	3,399564
22	2,369919	22	2,925261	22	3,603537
23	2,464716	23	3,071524	23	3,819750
24	2,563304	24	3,225100	24	4,048935
25	2,665836	25	3,386355	25	4,291871
26	2,772470	26	3,555673	26	4,549383
27	2,883369	27	3,733456	27	4,822346
28	2,998703	28	3,920129	28	5,111687
29	3,118651	29	4,116136	29	5,418388
30	3,243398	30	4,321942	30	5,743491
31	3,373133	31	4,538039	31	6,088101
32	3,508059	32	4,764941	32	6,453386
33	3,648381	33	5,003189	33	6,840509
34	3,794316	34	5,253348	34	7,251025
35	3,946089	35	5,516015	35	7,686087
36	4,103933	36	5,791816	36	8,147252
37	4,268090	37	6,081407	37	8,636087
38	4,438813	38	6,385477	38	9,154252
39	4,616366	39	6,704751	39	9,703507
40	4,801021	40	7,039989	40	10,285718
41	4,993061	41	7,391988	41	10,902861
42	5,192784	42	7,761588	42	11,557033
43	5,400495	43	8,149667	43	12,250455

The remaining calculations up to 50 years are omitted for want of room.

TABLE II.—*Showing the Amount of an Annuity of $1 or £1.*

Yrs	4 per cent.	Yrs	5 per cent.	Yrs.	6 per cent.
1	1,000000	1	1,000000	1	1,000000
2	2,040000	2	2,050000	2	2,060000
3	3,121600	3	3,152500	3	3,183600
4	4,246464	4	4,310125	4	4,374616
5	5,416322	5	5,525631	5	5,637092
6	6,632975	6	6,801913	6	6,975318
7	7,898294	7	8,142008	7	8,393837
8	9,214226	8	9,549109	8	9,897467
9	10,582795	9	11,026564	9	11,491315
10	12,006107	10	12,577892	10	13,180794
11	13,486351	11	14,206787	11	14,971642
12	15,025805	12	15,917126	12	16,869941
13	16,626837	13	17,712982	13	18,882136
14	18,291911	14	19,598631	14	21,015065
15	20,023587	15	21,578563	15	23,275968
16	21,824531	16	23,657491	16	25,672527
17	23,697512	17	25,840366	17	28,212879
18	25,645412	18	28,132385	18	30,905652
19	27,671229	19	30,539004	19	33,759991
20	29,778078	20	33,065954	20	36,785591
21	31,969201	21	35,719252	21	39,992727
22	34,247969	22	38,505214	22	43,392290
23	36,617888	23	41,430475	23	46,995827
24	39,082604	24	44,501999	24	50,815576
25	41,645908	25	47,727098	25	54,864512
26	44,311744	26	51,113454	26	59,156382
27	47,084214	27	54,669126	27	63,705765
28	49,967582	28	58,402582	28	68,528111
29	52,966286	29	62,322712	29	73,639798
30	56,084937	30	66,438847	30	79,058185
31	59,328335	31	70,760789	31	84,801677
32	62,701468	32	75,298829	32	90,889778
33	66,209527	33	80,063770	33	97,343164
34	69,857908	34	85,066959	34	104,183754
35	73,652224	35	90,320307	35	111,434779
36	77,598313	36	95,836324	36	119,120866
37	81,702246	37	101,628139	37	127,268118
38	85,970336	38	107,709545	38	135,904204
39	90,409149	39	114,095023	39	145,058456
40	95,025515	40	120,799774	40	154,761965
41	99,826536	41	127,889762	41	165,047683
42	104,819597	42	135,231751	42	175,950544
43	110,012381	43	142,993338	43	187,507577

TABLE III.—*Showing the present Value of an Annuity of $1 or £1.*

Yrs.	4 per cent.	Yrs.	5 per cent.	Yrs.	6 per cent.
1	,961538	1	,952381	1	,943396
2	1,886194	2	1,859410	2	1,833392
3	2,775190	3	2,723248	3	2,673011
4	3,629994	4	3,545950	4	3,465105
5	4,451821	5	4,329476	5	4,212363
6	5,242136	6	5,075691	6	4,917324
7	6,002054	7	5,786372	7	5,582381
8	6,732744	8	6,463211	8	6,209793
9	7,435331	9	7,107820	9	6,801691
10	8,110995	10	7,721733	10	7,360086
11	8,760576	11	8,306412	11	7,886874
12	9,385073	12	8,863249	12	8,383843
13	9,985647	13	9,393570	13	8,852682
14	10,563122	14	9,898638	14	9,294983
15	11,118487	15	10,379655	15	9,712248
16	11,652395	16	10,837767	16	10,105894
17	12,165668	17	11,274064	17	10,477258
18	12,659396	18	11,689585	18	10,827602
19	13,133938	19	12,085319	19	11,158115
20	13,590325	20	12,462208	20	11,469920
21	14,029159	21	12,821150	21	11,764075
22	14,451114	22	13,163000	22	12,041580
23	14,856840	23	13,488571	23	12,303377
24	15,246961	24	13,798639	24	12,550356
25	15,622078	25	14,093942	25	12,783355
26	15,982767	26	14,375183	26	13,003165
27	16,329584	27	14,643031	27	13,210533
28	16,663061	28	14,898125	28	13,406163
29	16,983712	29	15,141071	29	13,590720
30	17,292031	30	15,372448	30	13,764830
31	17,588491	31	15,592807	31	13,929085
32	17,873549	32	15,802673	32	14,084042
33	18,147643	33	16,002546	33	14,230228
34	18,411195	34	16,192901	34	14,368140
35	18,664610	35	16,374191	35	14,498246
36	18,908279	36	16,546848	36	14,620986
37	19,142576	37	16,711284	37	14,736779
38	19,367861	38	16,867889	38	14,846018
39	19,584482	39	17,017037	39	14,94907[illegible]
40	19,792771	40	17,159083	40	15,0462[illegible]
41	19,993049	41	17,294365	41	15,138015
42	20,185624	42	17,423205	42	15,224542
43	20,370792	43	17,545909	43	15,306172

A TABLE, *comparing the relative Weights of the principal trading Cities of Europe with those of America.*

lb.		lb.	oz.	
100	of England, Scotland, and Ireland, =	100	0	of America.
"	Amsterdam, Paris, Bourdeaux, &c.	109	8	
"	Antwerp, or Brabant, . .	103	12	
"	Rouen, ...	113	14	
"	City of Lyons, ...	94	3	
"	Rochelle, ...	110	9	
"	Toulouse and Languedoc, .	92	6	
"	Marseilles, ...	88	11	
"	Geneva, ...	123	0	
"	Hamburgh, ...	107	5	
"	Frankfort, ...	111	11	
"	Leipsic, ...	104	5	
"	Genoa, ...	73	0	
"	Leghorn, ...	75	8	
"	Milan, ...	65	3	
"	Venice, ...	65	11	
"	Naples, ...	64	10	
"	Seville and Cadiz, . . .	103	7	
"	Portugal, ...	95	4	
"	Leige, ...	104	0	
"	Spain, ...	97	0	

A TABLE, *comparing the American foot, of 12 inches, with the feet of various Countries in Europe.*

	in.	sec.	th'ds.		in.	sec.	th'ds.
London, ...	12	0	0	Mecklin, . . .	11	0	2
Antwerp, ...	11	4	1	Middleburg, . .	11	10	4
Bologna, . . .	14	5	2	France, ...	11	3	0
Bremen, . . .	11	6	4	Prague, ...	12	3	4
Cologne, . . .	11	5	2	Leyden, . . .	12	4	4
Copenhagen, . .	11	6	5	Riga, ...	21	11	3
Amsterdam, . .	11	3	4	Roman, . . .	11	7	2
Dantzic, ...	11	3	6	Scotch, ...	12	0	4
Dort, ...	14	2	3	Strasburgh, . .	11	0	2
Frankfort, . . .	11	4	3	Toledo, ...	10	9	3
Greek, ...	12	1	0	Turin, ...	12	8	6
Lorraine, . . .	11	5	6	Venice, ...	13	11	2
Mantua, ...	18	9	6				

TABLE OF CONTENTS.